Optimal Progressivity of Personal Income Tax
A General Equilibrium Evaluation for Spain

Master's Thesis

Department of Economics
Universität Mannheim

Submitted to: Prof. Krzysztof Pytka, Ph.D.

Submitted by: Darío Serrano Puente

Student ID: 1533417

Program: Master of Science in Economics (M.Sc.)

Address:
Calle Norte, 28
40260 Fuentepelayo, Segovia, Spain
☎ +34 626 91 22 97
✉ dserrano@mail.uni-mannheim.de

Mannheim, 27.01.2020

Abstract

Is the Spanish economy positioned at its optimal progressivity level in personal income tax? This article quantifies the aggregate, distributional, and welfare consequences of moving towards such an optimal level. A heterogeneous households general equilibrium model featuring both life cycle and dynastic elements is calibrated to replicate some characteristics of the Spanish economy and used to evaluate potential reforms of the tax and transfer system. The findings suggest that aggregate social welfare is maximized when both the level of progressivity and the pensions to non-workers are increased to some extent. In addition, this would bring more equality to the income distribution. However, not only are the effects on capital, labor, and output negative, but the level of wealth inequality increases, as the poorest households are discouraged from saving and working to a greater extent. Finally, these theoretical results are evaluated using tax micro data, which outlines a current scenario where the income-top households typically face suboptimal effective average tax rates.

1 Introduction

Many modern governments implement a redistributive fiscal policy, where income is taxed at an increasingly higher rate, while transfers tend to target the poorest households. For instance, in Spain there is a growing political debate taking up many headlines on the so-called "fiscal justice" that is putting on the table a tax rate increase for the high-income earners, i.e. an increase in the progressivity of the personal income tax.[1] This is not a minor issue, since most of the OECD economies obtain a large proportion of their tax collection through the taxation of personal income.[2] These policies are initially developed to produce a more egalitarian distribution of income and, consequently, to provide social insurance for both currently living households that suffer from large income fluctuations, and for future generations that face uncertainty about what their initial state will be.

Raising taxes on higher incomes may be potentially justified by the increase in income and wealth inequality in recent years, especially after the 2007 crisis. These concerns over rising economic inequalities have resulted in a huge body of literature. One of the clearest examples is the paper by Piketty (2015), which triggered a widespread discussion on the nature and evolution of wealth inequalities worldwide. A recent study by Anghel et al. (2018) provides an overview of the inequality levels in Spain and their evolution. They show that the wave of unemployment caused by the 2007 crisis resulted in a high inequality in per capita income.[3] As for the wealth inequality in Spain, they show that it exceeds income inequality and it increased after the crisis, which may be due to financial assets outperforming real assets according to their vision. By international standards, Spain's wealth inequality is moderate, as the ownership of real assets is more widespread than in other countries. Therefore, one is likely to consider that raising taxes on the income-rich could reverse the growing concentration at the top. However, in advanced market economies, such policies could be very costly in terms of efficiency (resource allocation).

The optimal design of a redistributive tax system is subject to many constraints. Bakis et al. (2015) emphasize three of them in their study about the transitional dynamics of setting an optimal progressivity level. First, agents may have access to insurance through other means (savings and bequests), then increasing the redistributiveness would alleviate the need for such self-insurance and crowd out capital accumulation, leading to reduced investment and output. Second, misinformation may prevent government from observing individual productivity and, by

[1]Besides, the personal income tax played an important role in the fiscal consolidation process of the Spanish economy, having been subjected to a large increase in 2012.

[2]For example, the average for the OECD countries of the share of personal income tax revenues over total tax revenues (excluding social security contributions) has been around 30-35% in recent years.

[3]Nevertheless, it is true that income inequality is lower when we take into account the household as a whole (due to insurance through inter-family transfers, pensions for the elderly, or young people delaying setting up home, among other potential factors).

levying taxes on total income, it could provoke incentive problems that discourage labor and thereby reduce output. Third, large-scale shifts in labor and capital supply (savings) alter the wage rate and the interest rate, which may have repercussions for income redistribution.

This is why having a quantitative theory that accounts accurately for the observed income and wealth inequality is crucial when assessing the aggregate, distributional, and welfare implications of certain policies. For that purpose, a heterogeneous households general equilibrium model economy is here calibrated to replicate some characteristics of the Spanish economy and used to compare the steady-state consequences of setting an optimal progressivity level in the Spanish personal income tax. For the Spanish case, general equilibrium models with heterogeneous agents have already been used to examine the effects of certain reforms.[4] However, when it comes to issues of inequality and distributions of wealth and income in Spain, not many studies are encountered to use general equilibrium models with heterogeneous agents to explore the relationship between fiscal policy variables and the endogenous cross-sectional distribution of income and wealth, in turn the main topic of analysis of this study. The references that we find with respect to this concern are mentioned in the following lines. Pijoan-Mas and González Torrabadella (2006) quantify the aggregate and distributional implications of an array of revenue neutral flat tax reforms for Spain. Viegas and Ribeiro (2015) attempt to characterize the Spanish debt consolidation process in order to assess its effects on economic inequality and welfare. And finally, Guner et al. (2018). This last work uses a life cycle model to evaluate a change in the progressivity of personal income tax in Spain, the same task that is addressed in this document. That is why the paper by Guner et al. (2018) is the clearest match to the study herein presented in terms of what both analyses discuss.

In general, the the literature on optimal taxation in a general equilibrium framework is vast, but these pieces of work do not have their focus on the Spanish context. Kindermann and Krueger (2018), Conesa et al. (2009), and Guner et al. (2017) study the effects of taxing higher incomes, i.e. top households, and particularly assess whether and to what extent capital should be taxed. Moreover, Bakis et al. (2015), Heathcote et al. (2017), and Conesa and Krueger (2006) provide an assessment of the optimal progressivity of personal income tax and how redistributive the government's fiscal policy should be. Finally, Díaz-Giménez and Pijoan-Mas (2019) evaluate the gains that a progressive consumption tax could have with the same modeling framework that is used in the present analysis. Hence, due to the topic and the underlying methodology, the work

[4] For example, Alonso-Borrego et al. (2005) build a general-equilibrium labor-search model with heterogeneous agents and firing costs for analyzing labor market reforms. Another case studies are Conde-Ruiz and González (2012) and Conde-Ruiz and González (2016), where they analyze Spain's 2011 pension reform and evaluate potential pension system scenarios à la Bismark or Beveridge for Spain. They use a model of heterogeneous agents with overlapping generations (OLG). Again, on the subject of demographic change and pension reform in Spain, two studies are found: Sánchez Martín (2010), which uses a general equilibrium model with heterogeneous agents and endogenous retirement, and the article by Sánchez Martín and Sánchez Marcos (2010), which uses a Two-Earners OLG general equilibrium model.

contained herein would contribute to this body of literature.

Again, although some previous studies have analyzed the Spanish economy with general equilibrium models devoted to study policy implications for wealth and income inequality, none of them has mixed the main characteristics of the dynastic and of the life cycle abstractions (hybrid model with retirement and bequests). Contrarily, these models are built in either dynastic or life cycle fashions. This is where this study adds value, as it proposes other methodologies previously used in other contexts to be applied to the Spanish scenario. The theoretical framework is built for Spain following Castañeda et al. (2003), who also rely on a hybrid approach to account for the U.S. earnings and wealth inequality.[5] Heterogeneity is introduced in this setup via distinct labor market opportunities using an uninsurable process on the endowment of efficiency labor units that features non-linear dynamics. Given the labor market opportunity, the households choose their work effort.[6] Life cycle characteristics are modeled using aging and retirement and dynastic links are modeled in a way that households are altruistic toward their descendants. Once the model is properly calibrated to match some empirical statistics of the Spanish economy, these features ensure that households save for precautionary motives (life cycle reasons and altruistic reasons).[7] This model economy replicates the distributions of income and wealth in very much detail. Further, it also works well when replicating the very top tails of those distributions.[8]

With the theoretical framework defined, to assess a bunch of progressivity reforms a different general equilibrium is calculated for each level of progressivity and pensions. Then a Benthamite social planner, who takes into account all households in the economy by putting the same weight on each of them, discerns the optimal progressivity reform. The findings suggest that aggregate social welfare is maximized when both the level of progressivity of the Spanish personal income tax and the pensions to non-workers are increased. More precisely, in the optimally reformed scenario, every household would increase by 1.57% its consumption in comparison to the ongoing scenario. By decomposing the aggregate welfare change, it is shown that most of the welfare

[5]This frames the model here presented within the methodological literature related to heterogeneous agents general equilibrium models with incomplete markets originally developed by Huggett (1993), Aiyagari (1994), Krusell and Smith (1998), Quadrini (2000), and De Nardi (2004), among others. These models (i) are devoted to account for income and wealth inequality and (ii) study decisions of households that face labor income processes that are random, household-specific, and uninsurable. In these model-based economies, households accumulate wealth in part to smooth their consumption.

[6]It means that the labor choice is set here to be endogenous (as in Pijoan-Mas and González Torrabadella (2006)).

[7]See Díaz-Giménez and Pijoan-Mas (2019) for a more detailed explanation.

[8]This is crucial for quantitative evaluations of tax reforms because the tax burdens and the incentives to work and save that a particular tax scheme creates are very different at different points of the income and wealth distributions, and they affect the most on the very income-rich and wealthy households. This distributional issues are relevant in measuring the trade-offs involved in choosing between efficiency and equality of tax reforms, since both aggregate and welfare changes depend on the number of households of each type that populate the economy.

gains are obtained by improvements in the tax and transfers system. On the contrary, the general equilibrium effects of the optimal reformed economy (related to equilibrium prices) and the effects resulting from the change in the equilibrium distribution of households generate welfare losses, but these losses together cannot overpass the welfare gains coming from the reformed tax and transfers system, resulting in aggregate welfare gains when taking into account every possible effect. These welfare gains are decomposed by household type, where it is observed that the non-workers or retirees are the ones who benefit the most from the reform, particularly when they are in the lower part of the income and wealth distributions. The workers are the ones who face the largest trade-off between (i) higher income due to an increased interest rate that pushes capital returns up and (ii) higher tax payments due to the increase in progressivity of the income tax. The effects of the measure on working households are those that lead to decreases in capital, labor, wages, and output. At the distributional level, this reform would reduce income inequality, but surprisingly increase wealth inequality as it discourages savings, mainly in the poorest households. Moreover, this study adds value in explaining the relationship between progressivity and aggregate and distributional measures. Finally, the theoretical results are evaluated with tax micro data. This last part shows how income-top households are facing effective average tax rates that are below what would be optimal from the point of view of a Benthamite social planner. Households between the 60th and the 80th percentiles would meet the same average tax rate in the actual than in the reformed scenario. However, households between the 20th and the 60th percentiles would experience a decrease in their average tax rates. More precisely, if the economy moves towards an optimal tax scheme, the average tax rate faced by a household between the 20th and the 40th percentiles would drop from 0.033 to 0.017, which is almost half of the actual value. On the other hand, households above the 80th percentile would experience a drastic increment in their average tax rate. For instance, the top 1% households of the household gross income distribution would go from confronting an average tax rate of 0.315 in the actual scenario to dealing with an average tax rate of 0.347 in the optimal scenario.

The remainder of the paper is structured as follows. The model is formally introduced in Section 2. Section 3 describes how the model has been calibrated to match the Spanish aggregate and distributional data. Section 4 presents the optimal reform of the progressivity based on a welfare comparison between steady-states and details the aggregate and distributional implications. Finally, Section 5 concludes.

2 The Model Economy

The model economy analyzed in this study, based on the setup proposed by Castañeda et al. (2003), is a modified version of the stochastic neoclassical growth model with uninsured idiosyncratic risk and no aggregate uncertainty. The main features of this theoretical framework can be summarized in as follows: (i) there is a number of households that are ex-ante identical (they all exhibit the same preferences); (ii) these households are differentiated among themselves by the uninsured household-specific shock that they receive in their endowments of efficiency labor units; (iii) households go through the life cycle and can be either workers or retirees (which can be interpreted as households out of the labor market in general); (iv) once households are retired, they face a probability of dying, and if they die, they are replaced by working-age descendants; and (v) households are altruistic towards their descendants.

2.1 Population and Endowment Dynamics

This particular model economy is inhabited by a measure one continuum of heterogeneous dynastic households. The households can be either of working-age or retired and they are all endowed with ℓ units of disposable time each period. Workers face an uninsured idiosyncratic stochastic process that determines their endowment of efficiency labor units. They also face an exogenous positive probability of retiring. Retired households are endowed with zero efficiency labor units and face en exogenous positive probability of dying. When a retired household dies, it is replaced by a working-age descendant that inherits the deceased household estate and, possibly, some of its earnings abilities. To denote the household's random age and random endowment efficiency labor units jointly, a one-dimensional shock, s, is used. This process is assumed to be *i.i.d.* across households and follows a finite state Markov chain with conditional transition probabilities given by $\Gamma_{\mathcal{S}\mathcal{S}'} = \Gamma\left(s'|s\right) = Pr\left\{s_{t+1} = s'|s_t = s\right\}$, where s and $s' \in \mathcal{S} = \{1, 2, \ldots, n\}$.

It is assumed that s takes values in one of two possible J-dimensional sets, $s \in \mathcal{S} = \mathcal{E} \cup \mathcal{R} = 1, 2, \ldots, J \cup J+1, J+2, \ldots, 2J$. When a household draws shock $s \in \mathcal{E}$, it is of working-age and endowed with $e(s) > 0$ efficiency labor units. When a household draws shock $s \in \mathcal{R}$, it is retired and endowed with zero efficiency labor units. When a household's shock changes from $s \in \mathcal{E}$ to $s' \in \mathcal{R}$, the household has retired. When it changes from $s \in \mathcal{R}$ to $s' \in \mathcal{E}$, the retired household dies and is replaced by a working-age descendant that inherits the estate, a_t, that the deceased household had at the end of period t. Therefore, the joint age and endowment process implies that the transition probability matrix $\Gamma_{\mathcal{S}\mathcal{S}'}$ controls (i) the demographics of the model economy by determining the expected durations of the households' working lives and retirements, (ii) the lifetime persistence of earnings by determining the mobility of households between states in $\mathcal{E}$, (iii) the life cycle pattern of earnings by determining how the endowments of efficiency labor units of new entrants differ from those of senior workers, and (iv) the intergenerational persistence of earnings by determining the correlation between the states in $\mathcal{E}$ for consecutive members of the same dynasty.

Since it is assumed that the presented joint age and endowment process takes values in two

J-dimensional sets, the number of realizations of such process is $2J$. Therefore, to specify the process on s, the values of $(2J)^2 + J$ parameters must be chosen. $(2J)^2$ of these parameters are the conditional transition probabilities and the remaining J are the values of the endowment of efficiency labor units. However, some assumptions about the nature of the joint age and endowment process impose some additional structure/restrictions on the transition probability matrix, $\Gamma_{\mathcal{S}\mathcal{S}'}$, which reduce the large number of parameters. In order to understand these restrictions better, it helps to consider the following partition of this matrix:

$$\Gamma_{\mathcal{S}\mathcal{S}'} = \begin{bmatrix} \Gamma_{\mathcal{E}\mathcal{E}} & \Gamma_{\mathcal{E}\mathcal{R}} \\ \Gamma_{\mathcal{R}\mathcal{E}} & \Gamma_{\mathcal{R}\mathcal{R}} \end{bmatrix} \tag{1}$$

Submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ describes the changes in the endowments of efficiency labor units of working-age households that are still of working age one period later, and no restrictions are placed on it so the values of J^2 parameters must be chosen. Submatrix $\Gamma_{\mathcal{E}\mathcal{R}}$ describes the transitions from the working-age states into the retirement states. This submatrix is defined by $\Gamma_{\mathcal{E}\mathcal{R}} := p_r\mathbf{I}$, where p_r is the probability of retiring and $\mathbf{I}$ is the identity matrix. This is because every working-age household faces the same probability of retiring and because only the last realization of the working-age shock is used to keep track of the earnings ability of the retirees. Consequently, the value of only one parameter must be chosen. Submatrix $\Gamma_{\mathcal{R}\mathcal{R}}$ describes the changes in the retirement states of retired households that are still retired one period later. This submatrix is defined by $\Gamma_{\mathcal{R}\mathcal{R}} := p_s\mathbf{I}$, where $(1 - p_s)$ is the probability of dying or exiting the economy. This is because the type of retired households never changes, and because every retiree faces the same probability of dying or exiting the economy. Therefore, the value of only one parameter is needed to identify this submatrix. Finally, Submatrix $\Gamma_{\mathcal{R}\mathcal{E}}$ describes the transitions from the retirement states into the working-age states that take place when a retired household dies and is replaced by its working-age descendant. The rows of this submatrix contain a two-parameter transformation of the stationary distribution of $s \in \mathcal{E}$, which is denoted by $\gamma^*_{\mathcal{E}}$. This transformation is aimed to control for both the life cycle profile of earnings and its intergenerational correlation. Intuitively, the transformation amounts to shifting the probability mass from $\gamma^*_{\mathcal{E}}$ toward both the first row of $\Gamma_{\mathcal{R}\mathcal{E}}$ and toward its diagonal.[9] Consequently, to characterize $\Gamma_{\mathcal{R}\mathcal{E}}$, one must choose the value of the two shifting parameters.

To keep the dimension of the process s as small as possible while still being able to achieve the calibration targets, a value of $J = 4$ is chosen. This means that $J^2 + J + 4 = 24$ parameters need to be chosen.[10]

[9] The definitions of the two shifting parameters can be found in the Section A of the Appendix. A detailed description of the mass shifting procedure can be found in Castañeda et al. (2003).

[10] $\Gamma_{\mathcal{S}\mathcal{S}'}$ has not yet been imposed to be a Markov matrix. When this is done, the number of free parameters is reduced to 20.

2.2 Preferences and Production Possibilities

Households are assumed to derive utility from consumption, $c_t \geq 0$, and from non-market uses of their disposable time. They also care about utility of their descendents as if it were their own utility. Consequently, households' preferences can be characterized by the following standard expected utility function:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c_t, \ell - h_t \right) | s_0 \right], \tag{2}$$

where the function u is continuous and strictly concave in both arguments, $0 < \beta < 1$ is the subjective-time discount factor, ℓ is the endowment of productive disposable time, and $0 \leq h_t \leq \ell$ is the labor choice. Consequently, $\ell - h_t$ is the amount of time allocated to non-market activities by the households.[11]

The functional form chosen for the households' common utility function is given by the following expression:

$$u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} + \chi \frac{(\ell - h)^{1-\varphi}}{1-\varphi} \tag{3}$$

This particular choice is widely used in this literature framework and is made because, as argued in Castañeda et al. (2003), the households in the model economy face very large changes in productivity, which, under standard nonseparable preferences, would result in extremely large variations of hours allocated to market activities. Aiming to avoid this, a more flexible functional form that is additively separable in consumption and leisure and that allows for different curvatures on these two variables is chosen. It implies that, to identify the households' preferences, five parameters (the four that identify the utility function and the subjective time discount factor) must be chosen.

On the other hand, as far as production possibilities are concerned, an aggregate product definition, Y_t, that depends on aggregate capital, K_t, and aggregate labor, L_t, is chosen.[12] This aggregate production function, $Y_t = f(K_t, L_t)$, exhibits constant returns to scale. Therefore, the choice for the particular functional form here is the Cobb-Douglas production function $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, where α is the capital share and the total factor productivity, A_t, is normalized to 1. Further, capital is assumed to depreciate geometrically at a constant rate, δ, and r and w are used to denote the prices of capital and of the efficiency units of labor before all taxes.[13] Therefore, to

[11] Note that retirees do not work, and consequently, they derive a constant utility from non-market activities.

[12] Aggregate capital is obtained by aggregating the wealth (asset position) of every household, and aggregate labor input is obtained by aggregating the efficiency labor units supplied by every household.

[13] The capital depreciation rate, δ, comes from the extensively used capital law of motion, i.e. $K_t = (1-\delta)K_{t-1} + I_t$, where I_t is the investment (namely, the fraction of income which is saved).

depict the aggregate technology, the values of two parameters, α and δ, must be chosen.

2.3 Government Sector

The government in the model economy taxes household income (from capital and from labor) and it uses the proceeds of taxation to make real transfers to retirees and to finance its consumption. Income taxes are described by the function $\tau(y_t)$, where y_t denotes the household income. Public transfers to retirees are described by the function $\omega(s_t)$.

Social security in this model economy takes the form of transfers to retirees that do not depend on past contributions made by households. Further, according to Conde-Ruiz and González (2012) and Conde-Ruiz and González (2016), the pension system in Spain seems to be very redistributive.[14] In general, lump-sum pensions are very redistributive, thus given the previous argument, a primary assumption that pension benefits are lump-sum could be a good approximation of the current system. These public pensions provide the non-working households with an insurance mechanism against the risk of living for too long, therefore it reduces their incentives to save or accumulate assets for precautionary motives, which makes it easier to replicate the fraction of households that own very few or zero assets. This setup allows for matching the size of average public retirement pensions paid in Spain, but it qualifies the precision of the analysis in two ways. First, the overall amount of idiosyncratic risk in the model economy diminishes because the labor market history does not condition the retirement benefits. Second, it abstracts from a potentially important reason to work, since in real world economies increasing the labor effort entitles the households to receive larger pension benefits.[15]

Therefore, a government policy rule in this model economy is a specification of $\{\tau(y_t),\, \omega(s_t)\}$ and of a process on government consumption, G_t. Since the government is running a budget balance scheme in every period, these policies must satisfy the condition $G_t + Tr_t = T_t$, where Tr_t and T_t denote aggregate transfers and aggregate tax revenues, respectively.

The household income taxes in the model economy are described by the function:

$$\tau(y) = \left[y - \lambda y^{1-\tau}\right] + \kappa y \tag{4}$$

The term within brackets is the function chosen by Heathcote et al. (2017) (HSV hereafter) to

[14] See their estimate of parameter α that measures the redistributiveness of the pension system.

[15] This assumption is made to reduce the computational complexity of the problem. Households receiving transfers according to their past contributions would require the inclusion of a second asset-type state variable in the household's decision problem. In addition, in a model with endogenous labor supply, linking pensions to contributions makes the optimality condition for leisure an intertemporal decision, which increases extremely the computational costs. See Díaz-Giménez and Pijoan-Mas (2019) for a more detailed explanation.

analyze the optimal progressivity of the tax system, the main purpose of this study.[16] This specification has been already used by García-Miralles et al. (2019) to replicate the Spanish income tax system.[17] Following their estimation strategy, the HSV is estimated here using an administrative dataset containing a (stratified) random sample of tax returns. This dataset is provided by the Spanish national tax agency (AEAT, its Spanish acronym) and contains a very detailed account of income from different sources, tax benefits, tax liabilities, and sociodemographic characteristics for a sample that accounts for a 14% of the population.[18] The fitting of the estimated HSV specification to the household data (for 2013, selected base year due to data availability to calibrate all parameters) is very precise, as shown in Figure 1. This justifies the choice of this specification for the model economy. The last term, κy is added to this tax function because the Spanish government obtains tax revenues from many other sources (property, estate, consumption, and excise taxes, among others), and this model economy abstracts from these tax sources.[19] Therefore, to specify the model economy household income tax function, a total of three parameter values must be chosen.

2.4 Market Arrangements

In this model economy, there are no insurance markets for the household-specific shock.[20] Instead, to buffer their streams of consumption against the shocks, households in the model economy can accumulate wealth in the form of real capital, $a_t \in \mathcal{A}$. The lower bound of the compact set $\mathcal{A}$ can be interpreted as a form of liquidity constraints, or as a solvency requirement (preventing

---

[16]In the HSV specification, λ determines the average taxes while τ determines the progressivity. When $\tau = 0$, taxes are flat and equal to $(1-\lambda)y$. When $\tau > 0$, taxes are positive, and higher levels of τ imply a greater degree of progressivity (the marginal rates exceed the average rates). When $\tau < 0$, the tax system is regressive (households with a non-negative income would obtain a net transfer from the government).

[17]See García-Miralles et al. (2019) for an extensive explanation of the application of the HSV tax function to calculate the after-tax income distribution for Spain with administrative data on tax returns. In the figure 8 of their study, they show how the HSV specification matches almost exactly the effective average tax rates for the Spanish households.

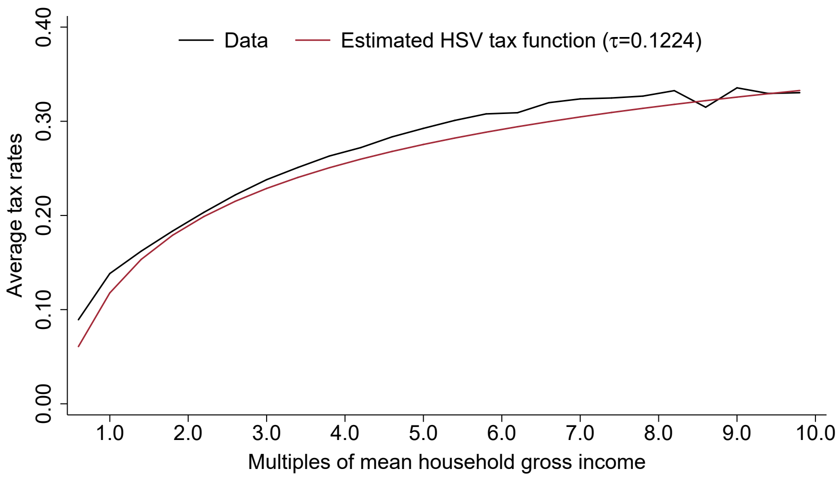
[18]The data is not censored either at the top or at the bottom of the income distribution. Recall that in Spain the income tax return can be filled in a single or in a joint (with the spouse) way, therefore, it is necessary to do aggregation in some cases to have the data at the household level.

[19]This choice implies that, in the model economy, it is assumed that all sources of tax revenues are proportional to income. It is equivalent to say that the government uses a proportional income tax to collect all the non-income tax revenues levied by the Spanish government. This type of augment of the tax function with a proportional term is widely used in the literature (e.g. see Castañeda et al. (2003) and Díaz-Giménez and Pijoan-Mas (2019)). Including this linear specification for remaining taxes could act as a consumption tax. This makes the choice of optimal progressivity more robust, since if consumption tax is not taken into account the optimal progressivity that the model finds can vary significantly, as argued by Guner et al. (2018).

[20]If insurance markets are allow to operate this economy, the model economy would collapse to a standard representative agent model.

Figure 1

Fitting of The HSV Specification to The Data



households that derive utility from leisure from going bankrupt).[21] Finally, firms are assumed to rent factors of production from households in competitive spot markets, which implies that factor prices are given by the corresponding marginal productivities.[22]

2.5 Households' Decision Problem

The individual state variables are the realization of the household-specific shock, s, and the value of the asset holdings, a.[23] The Bellman equation of the household decision problem is as follows:[24]

$$v(a,s) = \max_{c,a',h} u(c,\ell-h) + \beta \sum_{s\in\mathcal{S}} \Gamma_{\mathcal{S}\mathcal{S}'}\, v\left[a'(a,s),s'\right] \tag{5}$$

$$s.t. \qquad c + a' = y - \tau(y) + a \tag{6}$$

$$y = ar + e(s)hw + \omega(s) \tag{7}$$

$$\tau(y) = \left[y - \lambda y^{1-\tau}\right] + \kappa y \tag{8}$$

$$c \geq 0 \qquad a' \in \mathcal{A} \qquad 0 \leq h \leq \ell \tag{9}$$

where v is the households' common value function. Note that household income, y, includes three

[21]The existence of an upper bound for the asset holdings is guaranteed as long as the after-tax rate of return to savings is smaller than the households' common rate of time preference. This condition is always satisfied in equilibrium. See Huggett (1993), Aiyagari (1994), Ríos-Rull (1996), and Marcet et al. (2007) for details and proofs of this proposition.

[22]This means that $r = \alpha K^{\alpha-1}L^{1-\alpha} - \delta$ and $w = (1-\alpha)(\frac{r+\delta}{\alpha})^{\frac{\alpha}{\alpha-1}}$.

[23]There are no aggregate state variables since the model economy abstracts from aggregate uncertainty. The analysis is restricted to the steady state of the economy.

[24]Since the structure of the households' problem is recursive, henceforth the time subscripts are dropped from all the current-period variables, and primes are used to denote the value of variables one period ahead.

terms: capital income, ar, which can be earned by every household, labor income, $e(s)hw$, and retirement pensions, $\omega(s)$.[25] It is assumed that every household inherits the estate of the previous member of its dynasty at the beginning of the first period of its working life. More precisely, it is assumed that when a retiree exits the economy, it does so after that period's consumption and savings have taken place. Then, at the beginning of the next period, the deceased household's estate is liquidated and transmitted to the offspring.[26] Here is key that the asset choice for next period, a', that an agent makes at t is the household's stock wealth of its predecessor at the end of period t in the case that the predecessor is not going to be in the economy at period $t+1$. The household policy that solves this problem is a set of functions that map the individual state into the optimal choices for consumption, end-of-period savings, and labor hours. This policy is denoted by $\{c(a,s),\ a'(a,s),\ h(a,s)\}$.

2.6 Equilibrium

Each period the economy-wide state is a probability measure of households, x_t, defined over an appropriate family of subsets of $\{\mathcal{S} \times \mathcal{A}\}$ that counts the households of each type, and that is denoted by $\mathcal{B}$. In the steady state this measure is time-invariant, even though the individual state variables and the decisions of the individual households change from one period to the next one.[27]

Definition: A steady-state equilibrium for this economy is a household value function, $v(a,s)$; a household policy, $\{c(a,s),\ a'(a,s),\ h(a,s)\}$; a government policy, $\{\tau(y),\ \omega(s),\ G\}$; a stationary probability measure of households, x; factor prices, (r,w); and macroeconomic aggregates, $\{K,\ L,\ T,\ Tr\}$, such that:

(i) Given factor prices and the government policy, the household value function and the household policy solve the households' decision problem described in equations (5)-(9).

(ii) Firms behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities $r = f_K(K,L) - \delta$ and $w = f_L(K,L)$.

(iii) Factor inputs, tax revenues, and transfers are obtained by aggregating over households:

$$K = \int a \,\mathrm{d}x; \quad L = \int h(a,s)\, e(s)\, \mathrm{d}x; \quad Tr = \int \omega(s)\, \mathrm{d}x; \quad T = \int \tau(y)\, \mathrm{d}x$$

Every integral in the four definitions above is defined over the state space $\{\mathcal{S} \times \mathcal{A}\}$.

[25] Recall that $e(s) = 0$ when $s \in \mathcal{R}$ and $\omega(s) = 0$ when $s \in \mathcal{E}$.

[26] Note that this model economy abstracts from estate taxes levied by the government, i.e. there is no inheritance tax.

[27] See Huggett (1993) for further details.

(iv) The goods market clears:

$$\int \left\{ c(a,s) + a'(a,s) \right\} \mathrm{d}x + G = f(K,L) + (1-\delta)K$$

(v) The government budget constraint is satisfied: $G + Tr = T$.

(vi) The measure of households is stationary:

$$x(B) = \int_B \left\{ \int_{\mathcal{S}\times\mathcal{A}} a'(a,s)\, \Gamma_{\mathcal{S}\mathcal{S}'}\, \mathrm{d}x \right\} \mathrm{d}a'\mathrm{d}s'$$

for all $B \in \mathcal{B}$. This equation is aimed to count the households and to observe whether their asset holdings distribution is stationary. The procedure used to compute this equilibrium is in the Subsection C.4 of the Appendix.

3 Calibration

The model economy is characterized by 36 parameters: 5 for preferences, 2 for production technology, 4 for government policy, and 25 for the joint process on the age of the households and on the endowments of efficiency labor units (implied by the choice of $J = 4$ possible states during the working-age or the retiree phase of the life cycle).[28] To depict the values of these parameters, 36 calibration targets are needed. Of these targets, 7 are normalization conditions, and the remaining 29 are statistics that describe relevant features of the Spanish economy. Therefore, 29 target values describing the Spanish economy are needed. 7 of these calibration conditions uniquely determine the value of 7 model economy parameters. To determine the values of the remaining 22 parameters, we solve a system of 22 non-linear equations that results from equating the values of the model economy statistics to their empirical analogues in the Spanish economy. The details of the procedure used to solve this system can be found in Subection C.3 of the Appendix.

Model period. One important decision to be made is the length of the model period. It is set to be equal to one year since this is also the length of a tax period in Spain. Further, one year is also the length of the data collection period of the Survey of Household Finances wave used in this analysis, a survey conducted by the Banco de España that reports asset holdings at the household level for a representative sample of the Spanish population. Finally, due to data availability of this particular data source, 2013 is chosen as the calibration year.[29]

[28] See Section B of the Appendix for a full list of the parameters. There it is also stated which parameters are normalizations, which ones are directly identified, and which ones are estimated by means of the Simulated Method of Moments.

[29] Note that the last available data in the Survey of Household Finances corresponds to the 2014 wave, which accounts for the household finances during the previous year, 2013. Recall that this is the only available data on household balance sheets. It implies that, since the distributional data of wealth is needed for this analysis, the calibration year is conditioned by the availability of this data.

Normalization conditions. The household endowment of disposable time is an arbitrary constant and is chosen to be $\ell = 3.2$, as standard in related literature. The possible states in which a household can stay during its working-life or retirement are $J = 4$. In addition, the endowment of efficiency labor units of the least productive households is normalized to be $e(1) = 1$. Finally, since matrix Γ is a Markov matrix, its rows must add up to one. This property imposes four additional normalization conditions on the rows of $\Gamma_{\mathcal{E}\mathcal{E}}$.[30]

3.1 Macroeconomic Aggregates and Demographic Targets

Ratios. The target value for the capital to output ratio, K/Y, is 4.90 [Target 1], the capital income share, α, is 0.39 [Target 2], and the target value for the investment to output ratio, I/Y, is 20.27% [Target 3]. The target value for the capital to output ratio is obtained by dividing 273,579€, which was the average household net wealth in Spain in the calibration reference year according to OECD data, by 55,781€, which was per household Gross Domestic Product in Spain in the calibration reference year according to the Spanish National Institute of Statistics (*INE*, its acronym in Spanish).[31] The target value for the capital income share is obtained by subtracting the labor income share from a total measure of 1. The value for the labor income share was 0.61 in Spain in the calibration reference year, according to OECD data. Thus the capital income share, α is 0.39. To calculate the value of the target for I/Y, investment is defined as the sum of gross private fixed domestic investment, change in business inventories, and 75% of the private consumption expenditures in consumer durables using data for the calibration reference year from the Spanish National Institute of Statistics.[32] Further, note that the rate of depreciation of capital, δ, follows immediately from $\delta = I/K$ in stationary general equilibrium, hence $\delta = (I/Y)/(K/Y) = 0.0414$. These choices amounts to 3 targets.

Allocation of time and consumption. The average share of disposable time allocated to market activities is targeted to be $H/\ell = 33$ percent [Target 4]. This choice is standard in the literature and was already chosen to match its Spanish data counterpart.[33] For the curvature of consumption,

[30] The assumptions about the structure of the matrix Γ imply that once submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ has been appropriately normalized, every row of Γ adds up to one without imposing additional restrictions.

[31] This number of per household Gross Domestic Product is obtained by dividing the GDP by the number of households. This number of households is calculated by dividing the Spanish population quoted for the calibration reference year (46,766,403) by the average household size in Spain in the calibration reference year (2.53), both data according to the Spanish National Institute of Statistics.

[32] Since the National Accounts do not differentiate the amount associated with private consumption expenditures in durables and in non-durables, it is assumed that the durables share in the total reported private consumption expenditures is 5%.

[33] See Juster and Stafford (1991) for details about this number and Pijoan-Mas and González Torrabadella (2006) for an example of this choice to replicate the Spanish data counterpart.

a value of $\sigma = 1.5$ is chosen [Target 5].[34] This value falls within the range (1-3) that is standard in the literature.[35] These choices give two additional targets.

Age structure of the population. The expected durations of working-lives and retirement in the model economy are targeted to be 35 and 22.6 years [Targets 6 and 7], respectively, according to Eurostat data. These targets replicate the average durations of working-lives and retirement in Spain. These values serve as two more targets.

Life cycle profile of income. To replicate the life cycle profile of income in Spain in the model economy, the ratio of the average disposable income of agents between ages 41 and 65 to that of agents between ages 18 and 40 is the target value [Target 8].[36] In the 2006-2016 period, the average value of this statistic in Spain was 1.09, according to OECD data. This provides an additional target.

Intergenerational transmission of earnings ability. To replicate the intergenerational correlation of earnings of Spain in the model economy, the target is the cross-sectional correlation between the average lifetime earnings of one generation and the average lifetime earnings of their immediate descendents. Cervini Plá (2015) studies the intergenerational earnings and the income mobility in Spain and reports a value for this statistic of 0.42 [Target 9]. This translates into one more target value.

3.2 Government Policy

The parameters of the model economy household income tax are chosen so that the government collect the total tax revenues observed in the Spanish economy, which were 32.13% of the GDP in Spain in the calibration reference year, according to OECD data. In the model economy, these revenues must be entirely allocated to the expenditure side of the government. Therefore, since the government of the model economy must balance its budget, the output shares of government consumption, G/Y, and government transfers, Tr/Y (the two expenditure items in this model economy), are required to add up to 32.13%, which was the GDP share of total tax revenues, T/Y, in Spain in the calibration reference year, as previously mentioned. The target value for the transfers to output ratio in the model economy is 11.57% [Target 10], which corresponds to the share GDP accounted for by social security contributions in Spain in the calibration reference year, according to OECD data. This value is chosen so because, as discussed above, the social security transfers are very redistributive in Spain and a lump-sum transfer scheme may be a

[34] Recall that σ denotes the inverse elasticity of intertemporal substitution. Note that the CRRA preferences would collapse to a logarithmic functional form in the case that $\sigma = 1$.

[35] The calibration exercise by Pijoan-Mas (2006) reports a value of 1.46 and Heathcote et al. (2010) finds a value of 1.44. These calibration results finds very similar values for σ.

[36] Recall that the Spanish retirement age is (except to some particular cases) 65 years.

good approximation of the actual system, so all agents in the model economy receive a lump-sum transfer that, once it is aggregated, must be the same as the social security contributions that were paid by the working-age households. This choice means that the residual share for government expenditures to GDP amounts to 20.56% (= 32.13% − 11.57%) [Target 11], which is the target for the G/Y ratio in the model economy.[37]

Moreover, the model economy's income tax function is aimed to mimic the progressivity of the Spanish effective income taxes. This task was already done by García-Miralles et al. (2019) by means of the HSV specification. Therefore, to identify the tax function presented in equation 4, the values of parameters λ, τ, and κ must be chosen. Parameters λ and τ take the values estimated by García-Miralles et al. (2019) using administrative data on tax returns for Spanish households in 2013, which are 0.8823 and 0.1224 [Targets 12 and 13], respectively.[38] These two targets result from imposing that the shape of the model economy's tax function coincides with the shape of the function estimated by these authors and from assuming that all revenues levied from sources other than the Spanish income tax are proportional to income.

Finally, we just need to choose the values of the parameter ω, which stands for the lump-sum transfers to retirees, and the remaining parameter of the households' income tax function of this model economy that has not been already chosen, κ. More precisely, ω is chosen so as to achieve that the model economy transfers to output ratio, Tr/Y, mimics the value observed in the Spanish economy. On the other hand, the value of κ is chosen so that the government expenditures to output ratio in the model economy, G/Y, matches that observed amount in the Spanish economy. Note that the government in this model economy only allocates its spending on social security transfers and government spending. Then, because the two above parameters are chosen to match these two targets, it can also be said that the parameter κ is chosen so as to match the ratio of total tax revenues to output, T/Y, observed in the Spanish economy. This is equivalent to stating that the value of κ is selected such that the government in the model economy runs a budget balance policy in equilibrium, i.e. $G + Tr = T$.[39] These choices, in conjunction with those that mimic the shape of the model economy's tax function, represent 4 more targets.

[37]This is consistent with World Bank data for Spain for the year the calibration reference year, which present a general government final consumption expenditure (as % of GDP) of 20%.

[38]See their paper for a detailed description of the estimation process and the data used. They also estimate other specifications like the one established by Gouveia and Strauss (1994). Further, they propose an alternative method to estimate these specifications by differentiating between general income (labor and some capital gains, in the Spanish legislation) and capital income.

[39]Indeed, the value of this parameter is only completely calibrated when the general equilibrium of the economy is found, since such parameter acts as price solver of the general equilibrium by ensuring that the government runs a budget balance policy.

TABLE 1
STOCHASTIC PROCESS FOR THE ENDOWMENT OF EFFICIENCY LABOR UNITS

	$e(s)$	$\gamma_{\mathcal{E}}^*$	$\Gamma_{\mathcal{E}\mathcal{E}}$ from s to s'			
			$s'=1$	$s'=2$	$s'=3$	$s'=4$
$s=1$	1.00	11.29	96.10	3.80	0.09	0.01
$s=2$	1.68	31.41	0.99	98.31	0.68	0.02
$s=3$	3.86	56.81	0.21	0.02	99.55	0.22
$s=4$	95.40	0.49	2.72	17.69	6.23	73.37

Note: $e(s)$ denotes the relative endowment of efficiency labor units; $\gamma_{\mathcal{E}}^*$ denotes the stationary distribution of working-age households; $\Gamma_{\mathcal{E}\mathcal{E}}$ denotes the transition probabilities of the process on the endowment of efficiency labor units for working-age households that are still workers one period later.

3.3 Distributions of Income and Wealth

The aforementioned conditions specify a total of 20 targets (7 normalizations and 13 target values observed in the Spanish economy). To solve this model economy one must choose the value of 36 parameters. Therefore, 16 additional targets are added: the 2 Gini indexes and 14 additional points from the Lorenz curves of the Spanish distributions of income and wealth.[40] More precisely, the values for the off-diagonal elements of the submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ and for 3 out of 4 endowments of efficiency labor units are chosen so as to match (i) the Gini coefficients [Targets 14 and 15]; (ii) the income and wealth shares of the households concentrated between percentiles 1 to 40, 40 to 60, 60 to 80, and 80 to 100 [Targets 16 to 23]; and (iii) the income and wealth shares of the households concentrated between percentiles 90 to 95, 95 to 99, and the top 1% [Targets 24 to 29] of the distributions of income and wealth in Spain.[41] The 2013 household data to get the targets come from the 2014 wave (which corresponds to 2013 data) of the the Survey of Household Finances, a survey performed by the Banco de España, for wealth distributional statistics. As for the income distributional statistics, the 2013 household data comes from the already presented tax data provided by the Spanish tax agency.

3.4 Calibration Outcomes

The stochastic process of the endowment of efficiency labor units is characterized by the calibration procedure. This process, as shown in Table 1, presents strong skewness, fat right tail, and non-linear dynamics. It is important not to take this process literally, since it is an approximation that represents everything that is not known about the model economy.

[40]Similar calibration procedures can be found in Castañeda et al. (2003) and Díaz-Giménez and Pijoan-Mas (2019).

[41]Recall that the endowment of efficiency labor units of the least productive households is normalized to be $e(1) = 1.0$, thus the value of just 3 of these endowments must be chosen to match some characteristics of the Spanish income and wealth distributions.

TABLE 2

PARAMETER VALUES FOR THE BASELINE MODEL ECONOMY

Preferences		
Subjective time discount factor	β	0.9686
Curvature of consumption	σ	1.5000
Curvature of leisure	φ	2.0101
Relative share of consumption and leisure	χ	0.5000
Endowment of discretionary time	ℓ	3.2000
Technology		
Capital income share	α	0.3900
Capital depreciation rate	δ	0.0414
Age and endowment process		
Probability of retiring	p_r	0.0286
Probability of dying	$1 - p_s$	0.0442
Life cycle earnings profile	ϕ_1	0.9998
Intergenerational persistence of earnings	ϕ_2	0.9405
Fiscal policy		
Normalized transfer to retirees	ω	1.4155
Average level of income taxation	λ	0.8823
Progressivity of income taxation	τ	0.1224
Linear term on remaining taxes	κ	0.0524

The relative endowments of efficiency labor units are reported in the second column of Table 1 and the invariant measures of each type of working-age households are in the third column. The endowments of workers are calibrated such that the endowment of the least lucky (in terms of endowments of efficiency labor units) household is 1. Then, it can be observed that the luckiest workers in the model economy are 95 times as lucky as the unluckiest ones. The stationary distribution shows that each period 42% of the workers are very unlucky and draw states $s = 1$ or $s = 2$, while only one out of every 200 workers is extremely lucky and draws state $s = 4$. Finally, there is a group of households that are not that lucky in comparison with those households that draw state $s = 4$, but they are not as unlucky as the ones who draw states $s = 1$ or $s = 2$. These households that draw state $s = 3$ represents 56.81% of the invariant probability measure of households. The weight for these type of households is typically lower in the literature devoted to match the characteristics of the US economy, however, since the level of income inequality in Spain is much lower, this weight increases in order to generate an income distribution more concentrated in the not-top percentiles.[42]

The transition probabilities between the working-age states are reported in the last four columns of Table 1. The first three states are very persistent, while the last one is much less persistent. It can be observed that a worker whose current state is $s = 1$ is more likely to move to state $s = 2$ than to any of the other states. Likewise, a worker whose current state is $s = 2$ is most likely to move back to state $s = 1$. When the current state of a household is $s = 3$, it is more likely to move back not only to state $s = 2$ but also to $s = 1$ or to jump directly to state $s = 4$. Only very

[42] For the calibration of this endowment process in the US, see Castañeda et al. (2003) and Díaz-Giménez and Pijoan-Mas (2019), among others.

TABLE 3
BASELINE MODEL ECONOMY (BE) AND SPANISH ECONOMY (SPAIN)

Macroeconomic and fiscal ratios								
Economy	K/Y	I/Y	G/Y	T/Y	Tr/Y	h	$\rho_{o,y}$	$\rho_{f,s}$
Spain	4.90	20.27	20.56	32.13	11.57	33.00	1.09	0.42
BE	4.93	20.41	20.56	31.74	11.18	32.68	1.13	0.41
Distributional statistics								
Economy	Gini	Percentiles (%)				Top groups (%)		
		< 40	40-60	60-80	80-100	90-95	95-99	99-100
The distribution of income (before all taxes and after transfers)								
Spain	0.45	14.00	14.50	21.70	49.90	11.00	13.20	9.70
BE	0.41	16.10	14.53	21.00	45.34	10.40	12.31	10.25
The distribution of wealth								
Spain	0.68	3.54	9.83	18.19	68.44	12.81	19.82	20.02
BE	0.66	4.22	9.27	18.08	66.32	13.54	18.73	19.52

Note: h denotes the share of disposable time allocated to market activities; $\rho_{o,y}$ denotes the ratio of the average disposable income of agents between ages 41 and 65 (old) to that of agents between ages 18 and 40 (young); $\rho_{f,s}$ denotes the the cross-sectional correlation between the average lifetime earnings of one generation (fathers) and the average lifetime earnings of their immediate descendents (sons).

rarely workers whose current state is either $s = 1$ or $s = 2$ will make a transition to state $s = 4$. Finally, a household whose last draw was $s = 4$, it is most likely that it draws state $s = 2$ shortly after that draw in case that it does not remain in the same state.

The values of every other parameter of the model economy are presented in Table 2 and the statistics that describe the main aggregate and distributional features of the Spanish and the baseline model economies are reported in Table 3. These results confirm that overall the model economy succeeds in replicating the most relevant features of the Spanish economy in much detail. The ability of the model economy to replicate the macroeconomic ratios of the Spanish economy as well as the distributions of wealth and income can be appreciated. This result is particularly promising, since these two sets of targets are the main focus of analysis of this study.

4 Optimal Progressivity

This study aims to analyze what would be the optimal progressivity level of the personal income tax in Spain. Namely, it compares the current estimated level of progressivity ($\tau = 0.1224$) with a grid of different progressivity parameters ranging from 0.08 to 0.31. In every alternative scenario, the progressivity parameter is updated to a different value and a new general equilibrium of the model economy is calculated. It means that every reformed economy must be designed such that the markets clear (new interest rate, r, and new wage, w, are found) and the government balances its budget (the government chooses a new level of normalized transfers to non-workers, ω, associated with a new tax revenue derived from a new progressivity, τ). At this point, it is assumed that the government guarantees the same level of government expenditure to output, G/Y, as in the actual Spanish economy, i.e. 20,56%.[43][44] The gains or losses in tax revenue derived from each new level of progressivity are redistributed via lump-sum to households that are not present in the labor market. This particular choice of redistributiveness policy may be interpreted as helping households out of the labor market. In this model economy, the target beneficiaries of the reform are the retired households. Nonetheless, these households could represent any population group out of the labor market in the real economy, i.e. unemployed groups, groups with permanent limitations on accessing the workforce due to some reasons, and, of course, pensioners. This is particularly relevant due to the generational problem of the Spanish economy and the sustainability risk of the public pension system, since this evaluation could be framed within a possible case-study of financing pensions through the government budget (via collection of other taxes, such as personal income tax, instead of only financing them with social security contributions, which is the existing framework). As such, the optimal reform of income tax progressivity would be the one that maximizes the aggregate welfare gains.

4.1 Welfare Changes and Optimal Progressivity Level

Which level of progressivity from $\tau = 0.08$ to $\tau = 0.31$ results in a steady state with higher welfare?[45][46] Are these changes mainly driven by the change in the tax scheme? Or are they principally a result of induced shifts in the distribution of households or in the prices of the economy? How are these welfare changes distributed across households? A Benthamite social welfare function is used to answer these sort of concerns.[47]

[43] This choice is made so that the government always guarantees a level of expenditure over output (being output a measure of the size of the economy) implying that for each potential size of the economy the government would offer the same relative amount of education, health, etc.

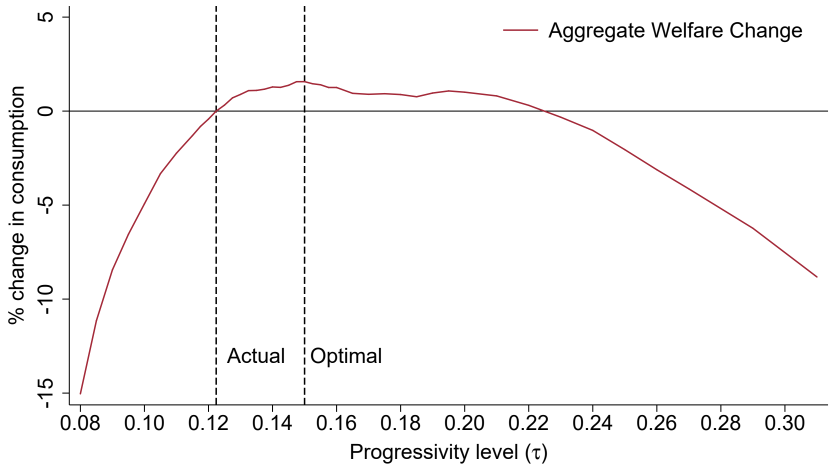
[44] See Appendix C.4.1 for a more involved explanation of the general equilibrium calculation in a reformed economy.

[45] Note that the choice of this grid for τ comes from the fact that (i) economies with $\tau < 0.08$ do not guarantee the same level of government expenditure to output as in the actual Spanish economy and (ii) economies with $\tau > 0.31$ present welfare loses. These results hail from evaluations of the model that are not here reported in order to focus only in the optimal part of the progressivity grid.

[46] The grid is not evenly split, but is a sparse grid with a higher concentration on its central values.

[47] Benthamite social welfare functions give identical weights to every household in the economy. Consequently, when the utility function is concave, equal sharing is the welfare-maximizing allocation. Notice also that the outcomes here presented emerge from a comparison between the welfare of different steady state allocations, but this study remains silent about the transitions between these steady states.

Figure 2
Aggregate Welfare Change



4.1.1 Aggregate Welfare Changes

To carry out the welfare comparisons, let $v_{BE}(a, s, \triangle)$ be the equilibrium value function of a household of type (a, s) in the baseline model economy, whose equilibrium consumption allocation is changed by a fraction $\triangle$ every period and whose leisure remains unchanged. Formally,

$$v_{BE}(a, s, \triangle) = u(c_{BE}(a, s)(1 + \triangle), \ell - h_{BE}(a, s)) + \beta \sum_{s \in \mathcal{S}} \Gamma_{\mathcal{S}\mathcal{S}'} \, v(a'_{BE}(a, s), s', \triangle) \tag{10}$$

where $c_{BE}(a, s)$, $h_{BE}(a, s)$, and $a'_{BE}(a, s)$ are the solutions to the households' decision problem defined in expressions (5) to (9). Next, the welfare gain of living in the steady state of a reformed economy, E_τ, for $\tau = \{0.08, ..., 0.31\}$, is defined as the fraction of additional consumption, $\triangle_\tau$, that must be given to, or taken away from, the households of the baseline economy so that the aggregate steady state welfare in the reformed economy E_τ is the same as in the baseline economy E_{BE}. Formally, $\triangle_\tau$ is the solution to the following equation:

$$\int v_{BE}(a, s, \triangle) \mathrm{d}x_{BE} = \int v_\tau(a, s) \mathrm{d}x_\tau, \tag{11}$$

where v_τ and x_τ are the equilibrium value function and the equilibrium stationary distribution of households in the reformed economy, E_τ.

The aggregate welfare gains or losses associated with each progressivity level are depicted in Figure 2. It can be here observed that any reform that increases the progressivity level from $\tau = 0.1224$ (the current level, or "actual level", these terms are used interchangeably) up to levels of $\tau = 0.22$ would result in aggregate welfare gains measured in percent changes in consumption of every household in the economy. In this regard, the optimal reform of the progressivity level of the

Spanish personal income tax would be found for $\tau = 0.15$. In such case, an equivalent variation in consumption of 1.57% would be found. This means that, from a Benthamite perspective, the steady state generated by a reform of implementing a progressivity level of $\tau = 0.15$ and increasing the transfers, ω, accordingly would be largely preferred to the steady state under the actual scenario. Indeed, the consumption of every household would need to be increased by 1.57% in every period and in every state for the social planner to be indifferent between the steady state allocation implied by the actual progressivity level and the steady state allocation that results from establishing a progressivity level of $\tau = 0.15$ and increasing the transfers, ω, accordingly.[48] Therefore, it can be deduced that implementing levels of progressivity lower than $\tau = 0.1224$ or higher than $\tau = 0.22$ would generate losses of aggregate welfare according to this setup.

4.1.2 A Decomposition of the Aggregate Welfare Changes

To improve the understanding of the results from a welfare change perspective, it is useful to decompose the equivalent variation in consumption previously presented. In order to do so, two additional measures of equivalent variations in consumption are defined. First, a particular equivalent variation in consumption is computed so that it makes the households indifferent between the baseline economy, E_{BE}, and the optimally reformed economy, $E_{0.15}$, ignoring the changes in the equilibrium distribution of households. Let $\triangle^a_{0.15}$ be such variation, which is defined as follows:

$$\int v_{BE}(a, s, \triangle^a_{0.15}; r_{BE}, w_{BE}) \mathrm{d}x_{BE} = \int v_{0.15}(a, s; r_{0.15}, w_{0.15}) \mathrm{d}x_{BE} \tag{12}$$

Note that the aggregate welfare of such hypothetical economy is calculated using the equilibrium price vector of the optimally reformed economy, $(r_{0.15}, w_{0.15})$, while the equilibrium stationary distribution is calculated using that of the baseline model economy, x_{BE}.

Second, another particular equivalent variation in consumption is computed so that it makes the households indifferent between the baseline model economy, E_{BE}, and the optimally reformed economy, $E_{0.15}$, ignoring both the changes in the equilibrium distributions of households and the changes in the size of the economy. Let $\triangle^b_{0.15}$ be such variation, which is defined as follows:

$$\int v_{BE}(a, s, \triangle^b_{0.15}; r_{BE}, w_{BE}) \mathrm{d}x_{BE} = \int v_{0.15}(a, s; r_{BE}, w_{BE}) \mathrm{d}x_{BE} \tag{13}$$

Note that the aggregate welfare of such hypothetical economy is now calculated using both the equilibrium stationary distribution, x_{BE}, and the equilibrium price vector, (r_{BE}, w_{BE}), of the baseline model economy.

[48] In this setup, the calibrated value for the normalized transfers to retirees, ω, would jump from 1.42 in the baseline economy to 1.97 in the optimally reformed economy.

Table 4

Decomposition of Aggregate Welfare Changes

Aggregate consumption equivalent variation	1.57%
Decomposition - Contributions to the aggregate welfare change by changes in:	
Taxes and transfers	441.45%
Equilibrium prices	-68.73%
Equilibrium distribution	-272.72%

Note: Each contribution to the aggregate welfare change is computed by dividing the consumption equivalent variation from changes in each factor by the aggregate consumption equivalent variation. Adding up three contributions makes one hundred percent.

These two additional equivalent variations in consumption allow for decomposing the total equivalent variation in consumption defined in expression (11) as follows:

$$\triangle_{0.15} = \triangle^{b}_{0.15} + (\triangle^{a}_{0.15} - \triangle^{b}_{0.15}) + (\triangle_{0.15} - \triangle^{a}_{0.15}) \tag{14}$$

The first term in equation (14) measures the welfare changes that are due to the reshuffling of resources between the households and it ignores both the general equilibrium effects of the optimal reform and the changes in the distribution of households, i.e. it measures the changes stemming from pure changes in the tax and transfer system. The second term shows the added welfare change triggered by changes in equilibrium prices, i.e. it measures the general equilibrium effects of the optimal reform. The last term measures the additional welfare change associated with changes in the equilibrium distribution of households.

The decomposition results are presented in Table 4. It is certainly interesting that most of the welfare gains are obtained by direct improvements in the tax and transfers system ($\Uparrow$). In contrast, the general equilibrium effects of the optimally reformed economy and the effects resulting from shifts in the equilibrium distribution of households show a welfare loss ($\Downarrow$), but these losses together cannot overpass the welfare gains coming from the reformed tax and transfers system, jointly resulting in aggregate welfare gains. The reform provokes a shrinkage in wages and an increment in the interest rate, both reactions making the welfare change associated with equilibrium prices be negative. When it comes to welfare changes coming from changes in the equilibrium distribution, these changes are negative, meaning that more households are concentrated in the lowest percentiles of the household distribution (ranked by value function), i.e. more households in the state space that have lower utility. However, despite these effects, the change in aggregate welfare is largely positive, showing how powerful such reform of the tax and transfer system is. This shows and reinforces the optimality of raising the progressivity level of the personal income tax in terms of aggregate welfare.

4.1.3 Welfare Changes by Household Types

The welfare improvements or detriments resulting from comparing the baseline model economy, E_{BE}, and the optimally reformed economy, $E_{0.15}$, are decomposed for different types of households in the spirit of Díaz-Giménez and Pijoan-Mas (2019). In the model economy there are as many households as there are $\{a, s\}$ pairs in the individual state space. To calculate the welfare changes at each point in the state space, $\triangle_{0.15}(a, s)$, the following equation is solved.

$$v_{BE}(a, s, \triangle_{0.15}(a, s)) = v_{0.15}(a, s) \tag{15}$$

The average of these individual welfare changes for various groups of households are reported in Figure 3.

Sorting by wealth. The households are here sorted by their asset holdings and the welfare gains are computed as an average over all the households, i.e. over all pairs of states (a, s), belonging to a certain group. The division of the wealth distribution leaves groups of households accounting for ten percentile points each. Looking at Panel A, it can be seen that all households benefit from the reform if we rank them by wealth. It can be discerned that households in the lower deciles benefit more from the reform, with decreasing welfare gains in wealth. There is a small exception with the poorest households in wealth, who curiously benefit the least. This may be because they are so poor that they do not accumulate any assets and have to work to be able to afford their consumption.[49] In this sense, their earned labor income (which is the only income they may have) is taxed at a higher rate as progressivity increases, which reduces their particular welfare gains. This narrative is clearly supported by what is shown in Panel B (welfare gains are reported for all four types of shock, s), where it is observed that workers in the first decile who have a large positive shock of efficiency are those who are most affected. It also shows how workers' welfare gains are decreasing in wealth and increasing in labor market opportunities. On the other hand, non-workers experience all welfare gains (see Panel C), with these gains being decreasing in wealth. This is because non-workers have all the same transfer from the government, ω, but capital gains, their other source of income, increase in wealth (due to a higher interest rate after the reform) and the higher capital income the more taxes they have to pay, especially due to the increase in the progressivity level. Therefore, the new income tax rate seems to reverse the former positive effect (higher capital returns derived from an increase in r) and makes welfare gains smaller the higher the level of asset accumulation is.

Sorting by income. The households are here sorted by their income before all taxes and after transfers. The most affected households are those at the center of the distribution (see Panel D),

[49] These are the households that are identified as poor-hand-to-mouth in the very influential study of Kaplan and Violante (2014).

FIGURE 3

WELFARE CHANGES BY TYPE OF HOUSEHOLD

Note: For the case of wealth sorting, states $s = 4$ for the percentiles 40-50 and 50-60 report welfare losses of the magnitude 38%, therefore these losses are not entirely reported in the figure since they escape from the axis limits. This is done for a better visualization of the plots.

since they are typically working households with little capital that suffer from the increase in the progressivity of the tax as they are basically fed by labor income, which is taxed at a higher rate under the reformed scenario. Households that are at the higher parts of the distribution also suffer from the reform, but somewhat less because they are compensated for the increase in what they pay in taxes by a higher income from capital due to a higher interest rate. For households in 7th decile, the increase in capital income outweighs the negative effect of the tax rate increase, thus achieving welfare gains from the reform. In addition, the other households benefiting from the reform are those in the first decile of the income distribution, driven mainly by the surpassing of the welfare increase of the poorest non-workers (resulting from the increase in ω) to the decrease of the poorest workers. These narrative can be realized in Panels E and F. The rest of non-workers (the ones in the 2nd and 3rd deciles) experience a decrease in welfare, which indicates that the negative tax-effect surpasses the positive income effect (higher capital gains due to increased interest rate and higher pension). It is particularly interesting that there are no retired households in the higher areas of the income distribution, which are solely occupied by workers. This is a reasonable finding since the endowment of efficiency labor units for these households is $(s) = 0$, thus not achieving any labor income. However, despite these households being concentrated at the bottom, they benefit from the reform through the upward update of their pension, ω, and the increase in the interest rate, which raises their capital returns, outweighing these effects the negative fallout derived from a higher tax rate.

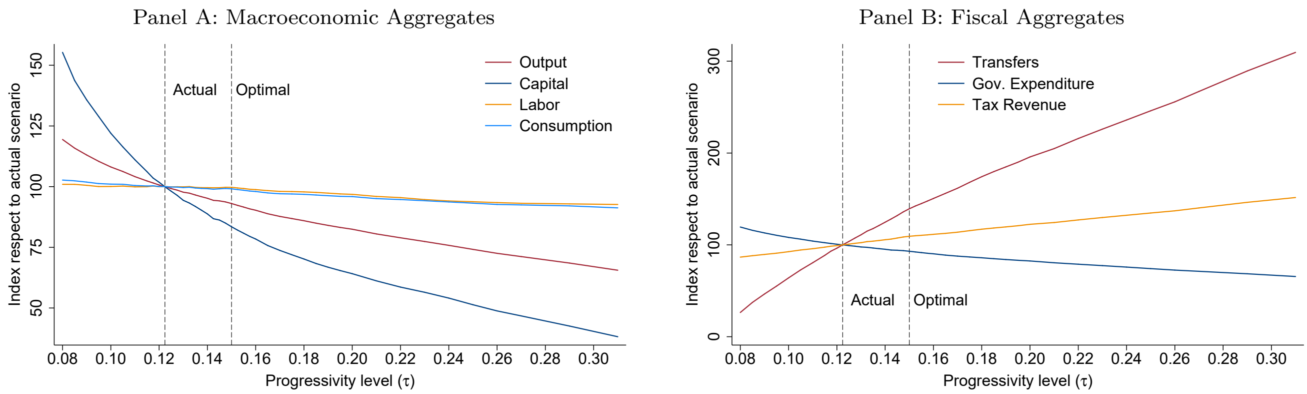
Sorting by value function. It is hard to determine who benefits more from the optimal reform sorting the households according to their wealth or according to their before-tax-after-transfers income. This is because permanent income is a function both of financial wealth and of human wealth. Alternatively, households are ranked by their value function, as it reflects their expected lifetime value given their individual state (a, s). The welfare gains are positive for almost all households and decrease in value function, as shown in Panel G. This pattern is mainly due to the behavior of the non-workers (see Panel I). Here one can more clearly discern how each and every non-working households benefit from the reform. This is due to the increased pension for all of them in the same way. However, those who benefit most are the asset-poor non-workers because they face less tax rates on capital proceeds. Finally, the effects for workers, once they have been ordered by value function, become less clear, although it could be said that the highest welfare gains are concentrated in the lower fifty percent of the distribution, with the lowest welfare gains (or even losses) in the upper hemisphere. This can be seen in Panel H. The difficulty to find a clear pattern for workers once they are sorted by value function (measure that takes into account indirectly both wealth and income) is a clear sign that workers are the ones who experience the largest trade-off between (i) positive welfare effects derived from higher income (due to an increased interest rate that pushes up capital returns) and (ii) adverse effects emerging from higher tax payments (due to the increase in progressivity of the income tax that discourages labor and savings).

4.2 Effects on Macroeconomic and Fiscal Aggregates

For each reform evaluated in the grid $\tau = \{0.08, ..., 0.31\}$, the main macroeconomic and fiscal aggregates are calculated. According to this, the evolution of these magnitudes on progressivity is depicted in Figure 4. The aim is to observe the behavior of these aggregates with respect to the progressivity of the personal income tax, assuming that the increase in tax collection will be redistributed via lump-sum to non-working households.

Broadly speaking, it is clear that aggregate capital and output are decreasing in progressivity in a convex pathway, with the drop in capital being more pronounced than in output. On the other hand, aggregate consumption and aggregate labor are also decreasing in progressivity. However, they do not fall in the same proportions as capital or output, but rather remain in a similar environment throughout the progressivity values grid. Another finding that is particularly curious is that the social planner, who determines which is the optimal reform, chooses the optimal level

FIGURE 4
THE MACROECONOMIC AND FISCAL AGGREGATES



Note: The actual scenario is base 100.

of τ at the exact point where the aggregate labor and consumption trends change (more precisely, where the fall of these aggregates increases somewhat more).

As for fiscal aggregates, due to the nature of the reforms assessed here, government expenditure falls in progressivity, since, as output falls, slightly less expenditure is needed to continue ensuring the G/Y level of the current scenario. In addition, transfers and tax revenue are increasing in progressivity in a convex manner, with a greater increase in the case of transfers.

The main macroeconomic and fiscal aggregates and ratios of the baseline economy, E_{BE}, and of the optimally reformed economy, $E_{0.15}$, are reported in Table 5. This is simply a comparison between the steady states of these two economies, which differ from each other in the parameter of progressivity, τ, and in the transfers to non-workers, ω. The percentage change in these magnitudes between these two economies is reported in the third row of the table.

Progressivity goes from $\tau = 0.1224$ to $\tau = 0.15$, which means that, in order for the government to maintain a balanced budget policy and a constant G/Y level, the additional revenue derived from the increase in progressivity will be allocated to non-working households in a lump-sum fashion, which makes the value ω go from 1.42 to 1.97. This fact causes wages, w, to fall from 1.69 to 1.58 and the interest rate, r, to go from 0.0377 to 0.0468. As shown in Table 5, increasing progressivity to its social optimum means that aggregate labor and the share of disposable time allocated to working activities are reduced. This is a directly derived result from an increasing taxation of labor income for the rich and some extra granted income is for the poor, which discourages labor and makes households opt to enlarge their leisure time, which they also derive utility from. This causes productivity per worker, denoted by Y/H, to fall. Likewise, aggregate capital falls, since the government, by taxing more the capital gains and by giving an extra lump-sum income to non-workers, is taking away precautionary motives for those agents to save. Thus, by lowering their savings rate, they accumulate less capital, which reduces investment on output, I/Y, and,

Table 5

Macroeconomic and Fiscal Aggregates and Ratios

Economy	Y	K	L^{*}	H^{**}/ℓ	K/L	L/H	Y/H	K/Y	I/Y	G/Y	T/Y	Tr/Y
E_{BE}	4.97	24.52	1.79	32.68	11.45	1.71	4.75	4.93	20.41	20.56	31.74	11.18
$E_{0.15}$	4.63	20.47	1.79	32.53	13.68	1.72	4.44	4.42	18.31	20.56	37.30	16.74
% change	-6.84	-16.52	-0.28	-0.47	19.47	0.19	-6.52	-10.28	-10.28	0.01	17.52	49.72

Note: * L denotes aggregate labor input; ** H/ℓ denotes the share of disposable time allocated to market activities.

consequently, aggregate capital, which induces the previously discussed rise in the interest rate. Further, aggregate output would also suffer a setback, since it is defined in terms of aggregate capital and aggregate labor, factors that, as mentioned above, fall when progressivity increases. Therefore, it can be interpreted that an increased tax collection (which finances the increase in transfers) enlarges the distortion in the intertemporal allocation of consumption, which encourages households to work less and save to a lower degree.

4.3 Effects on Income and Wealth Inequality

The Gini indexes and the Lorenz curves of income and wealth in the baseline economy, E_{BE}, and in the optimally reformed economy, $E_{0.15}$, are reported in Table 6. Here it can be realized how the increase in progressivity from $\tau = 0.1224$ to $\tau = 0.15$ entails an increment in wealth inequality and a decrease in income (before all taxes and after transfers) inequality, as shown by the Gini indexes.

The effect of the reform on the income distribution is not difficult to interpret. As income tax progressivity increases, households with a higher share of total income are those that suffer from the reform and those that experience a fall in their share, i.e. households beyond the 40th percentile lose about 4% of their share on average. However, the rest of the households in the distribution (the income-poor), witness how their share of total income increases by almost 20%. These results can by rationalized by a twofold effect: (i) lower aggregate income in the reformed scenario driving the share of the poorest households up and (ii) higher taxation of the richest households in favor of greater transfers for non-working households, which are the poorest in income. These findings show the strong redistributive power of the reform in terms of income distributional measures.

However, in terms of wealth distribution, the households with the lowest wealth are those that lose share of the total wealth to a wider extent. Households below the 40th percentile of wealth lose 68% of their already small share, while households between the 40th and 80th percentile experience a 20% drop in their share of total wealth after the reform. These consequences may be such because the reform discourages savings of the wealth-poor more than it does with the wealth-rich savings. Wealthy people, anticipating that they will be taxed more on their incomes (labor and capital gains), have less incentive than poorer people to lower their savings rates due to precautionary

Table 6
Distributions of Income and Wealth

Economy	Gini	Percentiles (%)				Top groups (%)		
		< 40	40-60	60-80	80-100	90-95	95-99	99-100
The distribution of income (before all taxes and after transfers)								
E_{BE}	0.41	16.10	14.53	21.00	45.34	10.40	12.31	10.25
$E_{0.15}$	0.36	19.26	13.80	20.06	43.75	10.43	11.76	9.74
% change	-11.80	19.61	-5.05	-4.50	-3.50	0.25	-4.49	-4.93
The distribution of wealth								
E_{BE}	0.66	4.22	9.27	18.08	66.32	13.54	18.73	19.52
$E_{0.15}$	0.73	1.36	7.37	14.29	74.88	15.74	23.59	18.80
% change	10.98	-67.68	-20.50	-20.99	12.91	16.24	25.97	-3.71

motives. This causes the distribution of wealth to stretch further, with the wealth-poor having less share and the wealth-rich having a larger share.

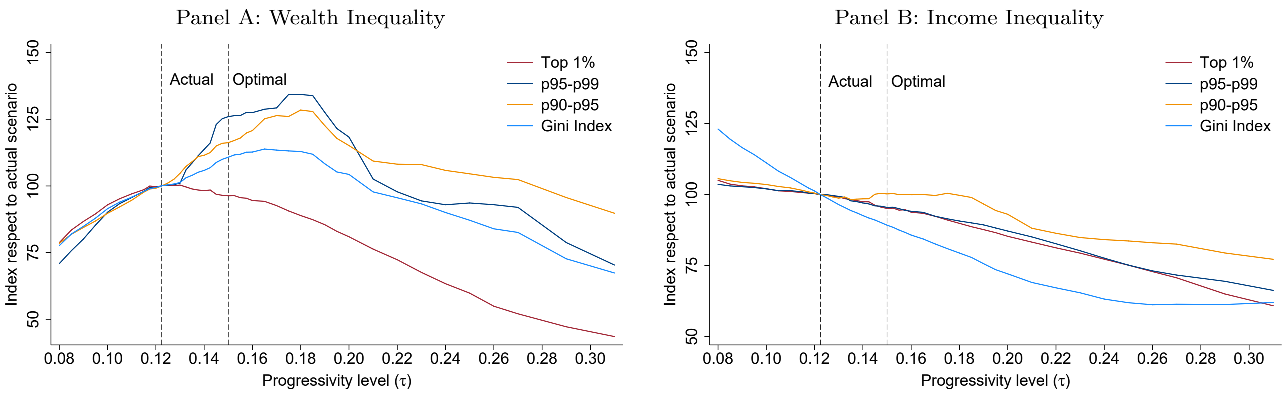
Further, in order to deepen the understanding of the relationship between income and wealth inequality and progressivity, it may be convenient to review Figure 5, where the levels of inequality are reported for each reform evaluated in the grid $\tau = \{0.08, ..., 0.31\}$.

In Panel A, looking at the relationship between measures of wealth inequality and income tax progressivity, one can see how the top 1% share, the 95-99 percentiles' share, the 90-95 percentiles' share, and the Gini index are increasing in progressivity in a similar way up to the actual value of progressivity ($\tau = 0.1224$). Above this level of progressivity, the evolution of each of these measures is non-trivial. While the share of the top 1% starts to decrease drastically in progressivity beyond the actual progressivity level, the shares of the 95-99 percentiles and the 90-95 percentiles increase, thus increasing the Gini index. These last measures do not start to decrease until a progressivity level of 0.19 is reached, and the Gini index in particular, in its decreasing trend, does not reach again levels of the actual scenario until it reaches a progressivity level of 0.21. This discovery would indicate that progressivity reforms reporting aggregate positive welfare changes would lead the households within percentiles 90 and 99 to increase their share of the total wealth. However, the top 1% share decreases because the households in this region of the distribution would suffer from an increase in the taxation in a way that they decide to lower their savings rate more than the households within percentiles 90 and 99, which suffer from the increase in taxes a bit less than the top 1% households.

When it comes to income inequality, it remains clear that its relationship with income tax progressivity is negative. More precisely, the top 1% and the 95-99 percentiles' shares of total income decrease in progressivity in a concave way along the complete τ grid. The Gini index follows also a downward trend in progressivity, but in a convex way. This is mainly due to the

FIGURE 5

INEQUALITY LEVELS



evolution of the 90-95 percentiles' share of total income, which is decreasing at a lower rate and even at some points increases with respect to the actual scenario. As an example, for the optimal progressivity level, $\tau = 0.15$, the share of the 90-95 percentiles increases by 0.25% with respect to the baseline economy, while the other top shares decrease, as previously shown in Table 6. This finding could indicate that, for this particular percentiles' share, the capital gains derived from a higher interest rate overpass the income losses produced by a higher taxation (due to an increased progressivity level).

4.4 Who Pays the Reform? Effects on the Personal Income Tax Scheme

Once the optimal progressivity reform has been analyzed in terms of welfare, macroeconomic and fiscal aggregates, and inequality, a relevant issue to approach is how this optimal level of progressivity would affect the Spanish taxpayers. For that purpose, making use of the aforementioned Spanish tax micro data, the potential impact of setting a progressivity level of $\tau = 0.15$ on the Spanish taxpayers is analyzed.

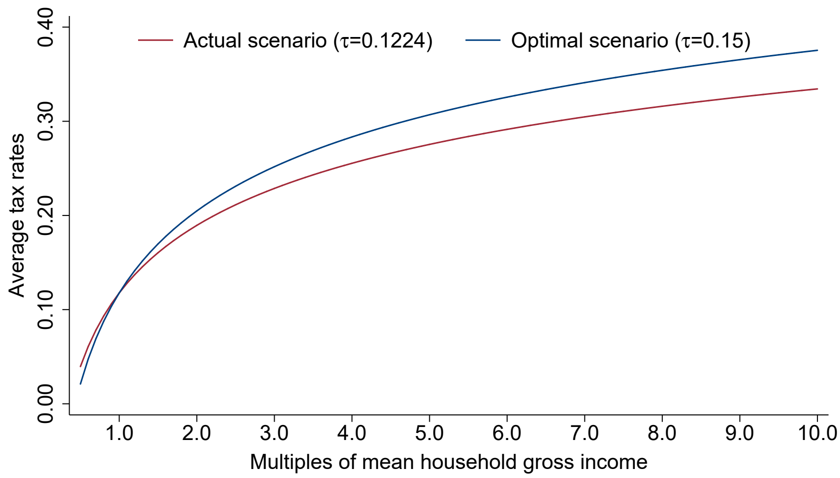
First, the average tax rate scheme for both the actual economy and the optimally reformed one is presented in Figure 6. Here, the HSV specification ($\bar{\tau} = 1 - \lambda \bar{y}^{-\tau}$, where $\bar{\tau}$ states for the average tax rate, $\bar{y}$ denotes the multiples of mean household gross income, and τ and λ are the estimated parameters that characterize the tax function of the model economy) is computed for the actual and the optimal scenario.[50][51] It can be realized how multiples of mean household gross income greater than 1 would experience an increase in the average tax rates under the optimal scenario, while multiples of mean household gross income lower than 1 would experience a decrease in the average tax rates. This is due to the nature of the reform, which changes the progressivity

[50]See Figure 1 to check the fitness of the HSV specification to the data.

[51]Note that for both lines the only changing parameter is τ. The parameter λ remains at the estimated level of 0.8823.

FIGURE 6

AVERAGE TAX RATE SCHEMES



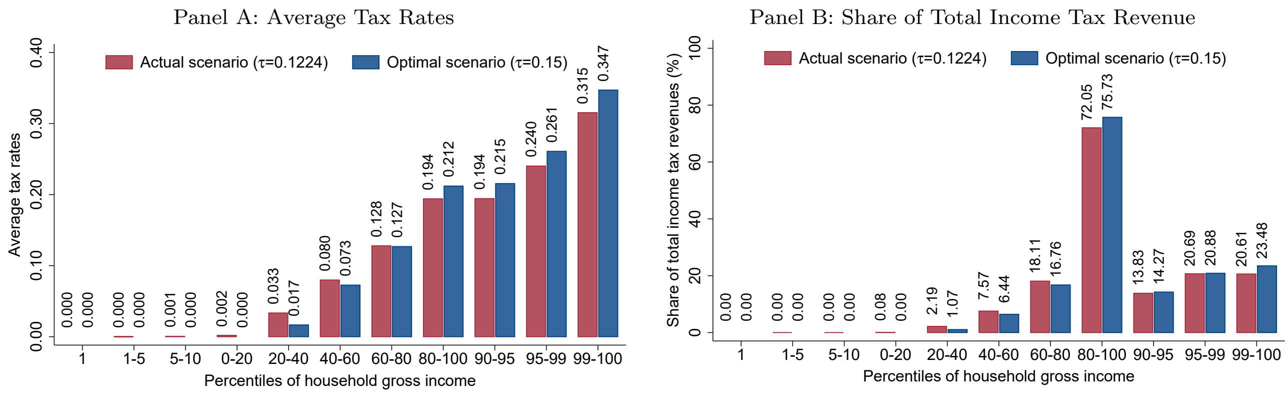
parameter τ, but leaves unchanged the parameter that controls for the average taxation in the economy, λ.

Further, following García-Miralles et al. (2019) for the estimation strategy of the HSV tax function, the effective average tax rates that every household face can be computed. In the Panel A of Figure 7, these effective average tax rates are reported by percentile groups of household gross income for the actual Spanish economy. Then, these numbers are compared with the average tax rates that the same households would face in the optimal scenario, i.e. setting $\tau = 0.15$.[52] Households in a region lower than the 20th percentile would continue to face very small or no average tax rates. Households between the 60th and the 80th percentiles would face the same average tax rate in the reformed scenario as in the actual one. However, households between the 20th and the 60th percentiles would experience a decrease in their average tax rates. More precisely, the average tax rate encountered by a household between the 20th and the 40th percentiles would drop from 0.033 to 0.017, which is almost half of the actual value. On the other hand, households above the 80th percentile would experience a drastic increment in their average tax rate. For instance, the top 1% households of the household gross income distribution would go from confronting an average tax rate of 0.315 in the actual scenario to dealing with an average tax rate of 0.347 in the optimal one. This would imply that the tax returns of households between the 60th and the 80th percentiles would account for the 75.73% of the total income tax revenue instead of the actual 72.05%. Particularly interesting is that the tax payments of the top 1% households of the household gross income distribution would account for 23.48% of the total income tax revenue under the optimal reform, an increase of 2.87 percentage points with respect to the actual scenario. These

[52]Note that this calculation abstracts from general equilibrium and distributional effects. This computation just involves computing the HSV specification for the average tax rates with $\tau = 0.15$ taking the disposable Spanish household gross income data as input of the specification.

Figure 7

Average Tax Rates and Share of Tax Revenue



results reveal that the optimal reform would be financed by the income-rich working households.

5 Concluding Remarks

In this paper, a heterogeneous households general equilibrium model featuring both life cycle and dynastic elements is calibrated to replicate some relevant characteristics of the Spanish economy and used to evaluate potential reforms of the tax and transfer system. Each of these reforms involves setting a different level of progressivity in the personal income tax and a different level of pensions to non-workers or retirees. This increase or decrease in the tax revenue, which is derived from moving towards a different level of progressivity, is redistributed to the non-workers via lump-sum transfers in a way that the government always balances its budget and guarantees the same level of public expenditure, G/Y.

The results of these evaluations reveal that elevating progressivity to a higher level than the current one could generate aggregate welfare gains. More precisely, the optimal reform of the progressivity level would be the one which maximizes aggregate welfare from the point of view of a Benthamite social planner. This planner takes into account all people in the economy in the same way. As a result of this welfare maximization setup, the optimal reform of the personal income tax would involve raising progressivity from 0.1224 (its current value) to 0.15. Such reform would induce that all households in the economy would increase their consumption by 1.57% each. By decomposing the aggregate welfare change, it is shown that most of the welfare gains are obtained by improvements in the tax and transfers system. Contrarily, the general equilibrium effects of the optimal reformed economy (higher interest rate and lower wage) and the effects resulting from changes in the equilibrium distribution of households (displacement to the lower areas of income) show a welfare loss, but these losses together cannot overpass the welfare gains coming from the reformed tax and transfers system, jointly resulting in aggregate welfare increments. In a next step, these welfare gains are decomposed by household type, where it is observed that the non-workers or retirees are the ones who benefit the most from the reform, particularly when they are in the lower part of the income and wealth distributions. In turn, the working households are those who experience the largest trade-off between (i) positive welfare effects derived from higher income (due to an increased interest rate that pushes up capital returns) and (ii) adverse effects emerging from higher tax payments (due to the increase in progressivity of the income tax that discourages labor and savings).

However, despite positive aggregate welfare effects, the consequences on aggregate capital, labor, and output would be negative, which means that the economy would experience an efficiency loss. Looking at the distributional implications, this reform would reduce income inequality, but surprisingly increase wealth inequality as it discourages savings, mainly in the poorest households. This leads one to think that taxing income to a higher degree can be counterproductive in tackling inequality issues. And this is where the role of wealth taxation emerges as a relevant element of analysis. Moreover, this study adds value in explaining the relationship between progressivity and aggregate and distributional measures, which sometimes is non-trivial. Finally, the theoretical results are evaluated with Spanish tax micro data. From the point of view of a Benthamite social planner, households between the 60th and the 80th percentiles would face the same average tax rate in the actual than in the reformed scenario. However, households between the 20th and the 60th percentiles would experience a decrease in their average tax rates under the optimal progressivity reform. To be more specific, the average tax rate encountered by a household between the 20th and the 40th percentiles would drop from 0.033 to 0.017, which is almost half of the actual value. On the other hand, households above the 80th percentile would experience a drastic increment in their average tax rate. For instance, the top 1% households of the household gross income distribution would go from confronting an average tax rate of 0.315 in the actual scenario to dealing with an average tax rate of 0.347 in the optimal scenario.

In conclusion, as policy implications arising from this study, what the model (jointly with the data) indicates is that, in terms of aggregate welfare, it would be optimal to increase the progressivity of the personal income tax and redistribute the increased tax revenue by targeting households out of the labor market. In addition, the reform would reduce income inequality. However, this would lead to an increase in wealth inequality and to a efficiency loss of the economy, since it discourages work and savings mainly penalizing working households. Therefore, assessing other taxes (such as wealth taxation) and focusing on the reform target (determining which households would receive the transfer and in which manner) emerge as additional central issues to be considered.

For future research there are several lines of investigation open. One of them would be to adapt the model so that it incorporates a specification for the budget constraint that parametrizes each tax separately (consumption tax, estate tax, social contributions, etc.). Another line would be to make the transfers or the retirement benefit progressive. In this way, other policies such as the universal basic income or the negative income tax could be evaluated. Beyond this, there is potential work to be done in finding transition paths between steady states, which could vary the optimal progressivity found by the model, as argued by Bakis et al. (2015). It would also be necessary to try to adapt the model to cover other types of methodological literature as well, which could make the results of the model more robust. For example, extend it to incorporate aggregate shocks to the economy (business cycles), to have heterogeneity in firms (some degree monopolistic power) or to introduce heterogeneity in the different marginal propensities to consume of individuals, which could be related to define different preferences for different households.

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 109, 659–684, https://doi.org/10.2307/2118417.

Alonso-Borrego, C., J. Fernández-Villaverde, and J. E. Galdón-Sánchez (2005): "Evaluating Labor Market Reforms: A General Equilibrium Approach," *NBER Working Paper*, 1159, https://doi.org/10.3386/w11519.

Andreasen, M. M. (2010): "How to Maximize the Likelihood Function for a DSGE Model," *Computational Economics*, 35, 127–154, https://doi.org/10.1007/s10614-009-9182-6.

Anghel, B., H. Basso, O. Bover, J. M. Casado, L. Hospido, M. Izquierdo, I. A. Kataryniuk, A. Lacuesta, J. M. Montero, and E. Vozmediano (2018): "Income, Consumption and Wealth Inequality in Spain," *SERIEs - Journal of the Spanish Economic Association*, 9, 351–378, https://doi.org/10.1007/s13209-018-0185-1.

Bakis, O., B. Kaymak, and M. Poschke (2015): "Transitional dynamics and the optimal progressivity of income redistribution," *Review of Economic Dynamics*, 18, 679–693, https://doi.org/10.1016/j.red.2014.08.004.

Castañeda, A., J. Díaz-Giménez, and J. V. Ríos-Rull (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111, 818–857, https://doi.org/10.1086/375382.

Cervini Plá, M. (2015): "Intergenerational Earnings and Income Mobility in Spain," *The Review of Income and Wealth*, 61, 812–828, https://doi.org/10.1111/roiw.12130.

Conde-Ruiz, J. I. and C. I. González (2012): "Spain 2011 Pension Reform," *FEDEA Documento De Trabajo*, https://doi.org/10.2139/ssrn.2077518.

——— (2016): "From Bismarck to Beveridge: the other pension reform in Spain," *SERIEs*, 7, 461–490, https://doi.org/10.1007/s13209-016-0148-3.

Conesa, J. C., S. Kitao, and D. Krueger (2009): "Taxing Capital? Not a Bad Idea After All!" *American Economic Review*, 99, 25–48, https://doi.org/10.1257/aer.99.1.25.

Conesa, J. C. and D. Krueger (2006): "On the optimal progressivity of the income tax code," *Journal of Monetary Economics*, 53, 1425–1450, https://doi.org/10.1016/j.jmoneco.2005.03.016.

De Nardi, M. (2004): "Wealth Inequality and Intergenerational Links," *The Review of Economic Studies*, 71, 743–768, https://doi.org/10.1111/j.1467-937x.2004.00302.x.

Díaz-Giménez, J. and J. Pijoan-Mas (2019): "Investment expensing and progressivity in flat-tax reforms," *SERIEs*, 1–35, https://doi.org/10.1007/s13209-019-0193-9.

García-Miralles, E., N. Guner, and R. Ramos (2019): "The Spanish Personal Income Tax: Facts and Parametric Estimates," *SERIEs*, 1–39, https://doi.org/10.1007/s13209-019-0197-5.

Gouveia, M. and R. P. Strauss (1994): "Effective Federal Individual Income Tax Functions: An Exploratory Analysis," *National Tax Journal*, 47, 317–339, https://dx.doi.org/10.1016/j.red.2014.01.003.

Guner, N., M. Lopez-Daneri, and G. Ventura (2017): "Heterogeneity and Government revenues: Higher taxes at the top?" *Journal of Monetary Economics*, 80, 69–85, https://doi.org/10.1016/j.jmoneco.2016.05.002.

Guner, N., J. López-Segovia, and R. Ramos (2018): "Reforming the Individual Income Tax in Spain," Mimeo.

Heathcote, J., K. Storesletten, and G. L. Violante (2010): "The Macroeconomic Implications of Rising Wage Inequality in the United States," *Journal of Political Economy*, 118, 681–722, https://doi.org/10.1086/656632.

——— (2017): "Optimal Tax Progressivity: An Analytical Framework," *The Quarterly Journal of Economics*, 132, 1693–1754, https://doi.org/10.1093/qje/qjx018.

Hopenhayn, H. A. and E. C. Prescott (1992): "Stochastic Monotonicity and Stationary Distributions for Dynamic Economies," *Econometrica*, 60, 1387–1406, https://doi.org/10.2307/2951526.

Huggett, M. (1993): "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 7, 953–969, https://doi.org/10.1016/0165-1889(93)90024-m.

Juster, F. T. and F. P. Stafford (1991): "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," *Journal of Economic Literature*, 29, 471–522, https://www.jstor.org/stable/2727521.

Kaplan, G. and G. L. Violante (2014): "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, 82, 1199–1239, https://doi.org/10.3982/ECTA10528.

Kindermann, F. and D. Krueger (2018): "High Marginal Tax Rates on the Top 1%? Lessons from a Life Cycle Model with Idio-syncratic Income Risk," *PIER Working Paper*, 14, https://doi.org/10.2139/ssrn.2509367.

Kirkby, R. (2017): "A Toolkit for Value Function Iteration," *Computational Economics*, 41, 1–15, https://doi.org/10.1007/s10614-015-9544-1.

Krusell, P. and A. A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896, https://doi.org/10.1086/250034.

Marcet, A., F. Obiols-Homs, and P. Weil (2007): "Incomplete markets, labor supply and capital accumulation," *Journal of Monetary Economics*, 54, 2621–2635, https://doi.org/10.1016/j.jmoneco.2006.12.011.

Pijoan-Mas, J. (2006): "Precautionary savings or working longer hours?" *Review of Economic Dynamics*, 9, 326–352, https://doi.org/10.1016/j.red.2005.11.002.

Pijoan-Mas, J. and M. González Torrabadella (2006): "Flat Tax Reforms: A General Equilibrium Evaluation for Spain," *Investigaciones Económicas*, 30, 317–351, https://dialnet.unirioja.es/servlet/articulo?codigo=1367629.

Piketty, T. (2015): "About Capital in the Twenty-First Century," *American Economic Review*, 105, 48–53, https://doi.org/10.1257/aer.p20151060.

Quadrini, V. (2000): "Entrepreneurship, Saving, and Social Mobility," *Review of Economic Dynamics*, 3, 1–40, https://doi.org/10.1006/redy.1999.0077.

Ríos-Rull, J. V. (1996): "Life-Cycle Economies and Aggregate Fluctuations," *Review of Economic Studies*, 63, 465–489, https://doi.org/10.2307/2297891.

Sánchez Martín, A. R. (2010): "Endogenous Retirement and Public Pension System Reform in Spain," *Economic Modelling*, 27, 336–349, https://doi.org/10.1016/j.econmod.2009.09.013.

Sánchez Martín, A. R. and V. Sánchez Marcos (2010): "Demographic Change and Pension Reform in Spain: An Assessment in a Two-earner, OLG Model," *Fiscal Studies*, 31, 405–452, https://doi.org/10.1111/j.1475-5890.2010.00120.x.

Viegas, M. and A. P. Ribeiro (2015): "Welfare and Inequality Effects of Debt Consolidation Processes: The Case of Spain, 1996-2007," *The Quarterly Journal of Economics*, 6, 479–496, https://doi.org/10.1007/s13209-015-0133-2.

A Transition Between Retirees and Descendants

A.1 Definitions of ϕ_1 and ϕ_2

The assumptions with respect to $\Gamma_{\mathcal{RE}}$ are dictated to evaluate the roles played by the life cycle profile of earnings and by the intergenerational transmission of earnings ability in accounting for income and wealth inequality. It turns out that these two roles can be modeled very parsimoniously using only two parameters.

Following the procedure presented in Castañeda et al. (2003), to determine the intergenerational persistence of earnings, the distribution from which the households draw the first shock of their working lives must be chosen. If this first shock is assumed to be drawn from the stationary distribution of $s \in \mathcal{E}$, which is denoted by $\gamma^*_{\mathcal{E}}$, then the intergenerational correlation of earnings will be very small. In contrast, if it is assumed that every working-age household inherits the endowment of efficiency labor units that its predecessor had at the end of its working life, then the intergenerational correlation of earnings will be relatively large. Since the target value for this correlation, 0.42, lies between these two extremes, an additional parameter, ϕ_1, is needed to act as a weight that averages between a matrix with $\gamma^*_{\mathcal{E}}$ in every row, denoted by $\Gamma^*_{\mathcal{RE}}$, and the identity matrix, $\mathbf{I}$. Intuitively, the role played by this parameter is to shift the probability mass of $\Gamma^*_{\mathcal{RE}}$ towards its diagonal.

Further, to assess the life cycle profile of income, the defined target is the ratio of the average income of agents at the end of their working lives (41-65 years old) to that of the agents at the beginning of their working lives (18-40 years old). The value of this statistic in the presented model economy is determined by differences in earnings ability between new entrants and senior workers. If it is assumed that every new entrant starts its working stage with a shock drawn from $\gamma^*_{\mathcal{E}}$, then the household earnings will be independent of household age. In contrast, if every household starts its working life with the smallest endowment of efficiency labor units, then the household earnings will grow too fast with household age. Since the target value for the life cycle earnings ratio lies between these two extremes, 1.09, an additional parameter, ϕ_2, is needed to act as a weight that averages between $\Gamma^*_{\mathcal{RE}}$ and a matrix with a unity vector in its first column and zeros elsewhere. Intuitively, the role played by this second parameter is to shift the probability mass of $\Gamma^*_{\mathcal{RE}}$ towards its first column.

Unfortunately, the effects of ϕ_1 and ϕ_2 on the two statistics of interest previously mentioned are of opposite sign. Consequently, this parsimonious modeling strategy might not be flexible enough to attain every desired pair of values for the targeted statistics.

A.2 Transition Submatrix Formula

This Subsection explains how $\Gamma_{\mathcal{RE}}$ is computed and how ϕ_1 and ϕ_2 affect such transition submatrix. Let p_{ij} denote the transition probability from $i \in \mathcal{R}$ to $j \in \mathcal{E}$, let γ_i^* be the invariant measure of households that receive shock $i \in \mathcal{E}$, and let ϕ_1 and ϕ_2 be the two parameters that shift the probability mass towards the diagonal and towards the first column of the submatrix $\Gamma_{\mathcal{RE}}$, then the recursive procedure that is used to compute the p_{ij} is the following:

Step 1: First, ϕ_1 is used to shift the probability mass from a matrix with vector $\gamma_{\mathcal{E}}^* = (\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*)$ in every row towards its diagonal, as follows:

$$
\begin{aligned}
p_{51} &= \gamma_1^* + \phi_1\gamma_2^* + \phi_1^2\gamma_3^* + \phi_1^3\gamma_4^* \\
p_{52} &= (1-\phi_1)[\gamma_2^* + \phi_1\gamma_3^* + \phi_1^2\gamma_4^*] \\
p_{53} &= (1-\phi_1)[\gamma_3^* + \phi_1\gamma_4^*] \\
p_{54} &= (1-\phi_1)\gamma_4^* \\
p_{61} &= (1-\phi_1)\gamma_1^* \\
p_{62} &= \phi_1\gamma_1^* + \gamma_2^* + \phi_1\gamma_3^* + \phi_1^2\gamma_4^* \\
p_{63} &= (1-\phi_1)[\gamma_3^* + \phi_1\gamma_4^*] \\
p_{64} &= (1-\phi_1)\gamma_4^* \\
p_{71} &= (1-\phi_1)\gamma_1^* \\
p_{72} &= (1-\phi_1)[\phi_1\gamma_1^* + \gamma_2^*] \\
p_{73} &= \phi_1^2\gamma_1^* + \phi_1\gamma_2^* + \gamma_3^* + \phi_1\gamma_4^* \\
p_{74} &= (1-\phi_1)\gamma_4^* \\
p_{81} &= (1-\phi_1)\gamma_1^* \\
p_{82} &= (1-\phi_1)[\phi_1\gamma_1^* + \gamma_2^*] \\
p_{83} &= (1-\phi_1)[\phi_1^2\gamma_1^* + \phi_1\gamma_2^* + \gamma_3^*] \\
p_{84} &= \phi_1^3\gamma_1^* + \phi_2^2\gamma_2^* + \phi_1\gamma_3^* + \gamma_4^*
\end{aligned}
$$

Step 2: Then for $i = 5, 6, 7, 8$, parameter ϕ_2 is used to shift the resulting probability mass towards the first column as follows:

$$
\begin{aligned}
p_{i1} &= p_{i1} + \phi_2 p_{i2} + \phi_2^2 p_{i3} + \phi_2^3 p i_{i4} \\
p_{i2} &= (1-\phi_2)[p_{i2} + \phi_2 p_{i3} + \phi_2^2 p_{i4}] \\
p_{i3} &= (1-\phi_2)[p_{i3} + \phi_2 p_{i4}] \\
p_{i4} &= (1-\phi_2)p_{i4}
\end{aligned}
$$

TABLE 7

STOCHASTIC PROCESS FOR THE ENDOWMENT OF EFFICIENCY LABOR UNITS AND AGE

	$e(s)$	γ^*	$\Gamma_{\mathcal{SS}}$ from s to s'							
			$s'=1$	$s'=2$	$s'=3$	$s'=4$	$s'=5$	$s'=6$	$s'=7$	$s'=8$
$s=1$	1.00	29.02	93.35	3.69	0.09	0.01	2.86	0.00	0.00	0.00
$s=2$	1.68	25.30	0.96	95.50	0.66	0.02	0.00	2.86	0.00	0.00
$s=3$	3.86	6.33	0.20	0.02	96.71	0.21	0.00	0.00	2.86	0.00
$s=4$	95.40	0.07	2.64	17.18	6.05	71.27	0.00	0.00	0.00	2.86
$s=5$	0.00	18.78	4.42	0.00	0.00	0.00	95.58	0.00	0.00	0.00
$s=6$	0.00	16.37	4.16	0.26	0.00	0.00	0.00	95.58	0.00	0.00
$s=7$	0.00	4.09	3.91	0.25	0.26	0.00	0.00	0.00	95.58	0.00
$s=8$	0.00	0.04	3.68	0.23	0.25	0.26	0.00	0.00	0.00	95.58

Note: $e(s)$ denotes the relative endowments of efficiency labor units; γ^* denotes the stationary distribution households; $\Gamma_{\mathcal{SS}}$ denotes the transition probabilities of the process on the endowment of efficiency labor units and age.

A.3 Joint Age and Endowment Stochastic Process

The transition matrix between all possible states of the model economy is presented in Table 7. This is the Markov Chain transition matrix featuring the stochastic process that jointly defines the age and the endowment of labor efficiency units of each household. A more detailed definition of each of the sub-matrices found in this matrix can be reviewed in the Subsection 2.1.

B Full list of parameters

The setup of this model economy has 36 parameters. A full description of the parameters and how they are calibrated or estimated is contained in the body of the paper. For convenience, a complete list of parameters is provided here. It is also indicated which ones are normalizations, calibrated, or estimated.

A full list of the parameters:

5 parameters to describe the household's preferences

β, σ, φ, χ, ℓ

2 parameters for production technology

α, δ

4 parameters for the government policy

ω, λ, τ, κ

25 parameters for the joint process on age and endowment of efficiency labor units

J, p_r, p_s, ϕ_1, ϕ_2

5 so far, and 20 more for the submatrix $\Gamma_{\mathcal{EE}}$ and the endowments of efficiency labor units

$e(s) = [e(1), e(2), e(3), e(4), 0, 0, 0, 0]$

(4 here: $e(1), e(2), e(3), e(4)$)

$\Gamma_{\mathcal{E}\mathcal{E}} = [\gamma_{11}, \gamma_{12}, \gamma_{13}, \gamma_{14}; \gamma_{21}, \gamma_{22}, \gamma_{23}, \gamma_{24}; \gamma_{31}, \gamma_{32}, \gamma_{33}, \gamma_{34}; \gamma_{41}, \gamma_{42}, \gamma_{43}, \gamma_{44}]$
(and 16 here)

Normalization, 7 parameters: $J = 4$, $\ell = 3.2$, $e(1) = 1$ (endowment for the least productive households), and four normalizations on the rows of $\Gamma_{\mathcal{E}\mathcal{E}}$ (so that each rows adds up to one).

Directly Identified, 7 parameters: σ, λ, τ, p_r, p_s, α, δ

Estimated by Simulated Method of Moments, 22 parameters:
Those for preferences, government, and taxation: β, φ, χ, ω, κ
And 17 parameters for the joint process on age and endowments of efficiency labor units: ϕ_1, ϕ_2, $e(s) = [e(1), e(2), e(3), e(4), 0, 0, 0, 0]$ (3 here, since $e(1) = 1$ is a normalization of the least productive household)
and $\Gamma_{\mathcal{E}\mathcal{E}} = [\gamma_{11}, \gamma_{12}, \gamma_{13}, \gamma_{14}; \gamma_{21}, \gamma_{22}, \gamma_{23}, \gamma_{24}; \gamma_{31}, \gamma_{32}, \gamma_{33}, \gamma_{34}; \gamma_{41}, \gamma_{42}, \gamma_{43}, \gamma_{44}]$
(12 here, which are the off-diagonal elements, as the four normalizations must be done for the diagonal elements)

Note that the choice to use the diagonal elements of $\Gamma_{\mathcal{E}\mathcal{E}}$ as the elements to be normalized has an important advantage in computation. Since the off-diagonal elements are smaller, by having the diagonal elements given by whatever was leftover to make the row sum up to one, the problem that they may end up being negative is avoided.

C Computation

This Section describes the computation of the model. First, it describes how the value functions and the stationary distribution are computed. Second, it provides a brief description of how the model moments are calculated. Third, it sheds some light on the calibration of such model economy. Finally, it clarifies the process to get the general equilibrium of the model. It is particularly important to mention that the codes are written and executed in Matlab in the spirit of the Value Function Iteration algorithm developed by Kirkby (2017).

All simulation exercises involved a burn-in of 1000 points (typically starting from the 'mid-point' of the relevant distribution).

C.1 Value Functions and Stationary Distribution

To calculate the optimal decision rules, the state space is discretized and a value function iteration using the Howard's improvement algorithm is performed.

The size of the state space is $n_a \times n_s = 681 \times 8 = 5,448$ points. The size of the control space is $n_a \times n_h = 681 \times 7 = 4,767$ points. Since the numbers of working-age and retirement states are $n_{\mathcal{E}} = n_{\mathcal{R}} = 4$, the total number of search points is $[(n_a \times (n_{\mathcal{E}} + n_{\mathcal{R}})) \times (n_a \times_h)] = 25,970,616$.

The stationary distribution is approximated with a discretization of the associated distribution function. The grid for this approximation is the same as that used to solve for the value function. The stationary distribution is calculated by iterating on the whole distribution, using the optimal policy functions and the transition matrix of the idiosyncratic shocks (exogenous process that determines the age and the endowment of efficiency labor units of the households). This is done until it converges, as measured by a distance criterion based directly on the monotone mixing condition underlying the theory that ensures that a stationary distribution exists.[53][54] This process is more demanding computationally than those typically used for calculating the stationary distribution, but that is important since the model moments relating to the top percentiles, e.g. the asset share of the top decile of asset holders, otherwise varied substantially between different simulations (much less simulations were fine for giving stable results for the first moments, such as the capital stocks, but the top percentile moments can be quite volatile).

C.2 Model Moments

The distributional and aggregate moments or statistics of the model economy can almost all be computed directly as integrals with respect to the stationary distribution of households. It means that this just involves taking weighted sums, as the stationary distribution is approximated as a weight for each point on a grid. The only exceptions are those moments that measure the earnings life cycle profile and the intergenerational correlation of earnings. These particular computations are presented in what follows straight after.

Life cycle profile of income. This is measured as the ratio of the average disposable income of agents between ages 41 and 65 to that of agents between ages 18 and 40. To compute this particular measure, a random newborn is drawn from the distribution of newborns in a first step.[55] Then, the household is simulated for 30 periods recording its productivity in each year and recording both its average earnings between ages 18 and 40 and those between ages 41 and 65. It is done for a large number of households and, after dropping those ones retired before reaching the old phase,

[53] See Hopenhayn and Prescott (1992).

[54] To speed up the convergence, this process is started by iterating on the stationary distribution from an initial distribution created by 1,000,000-points simulation. Actually, to take advantage of parallelization, this was implemented as *ncore* simulations of 1,000,000/*ncore* points each, where *ncore*=4 was the number of cores in the computer where the algorithm was executed.

[55] It is implemented as drawing a random retired household, forcibly killing it, and then determining where it would end up as a new born. Since the probability of exiting the economy is equal for all retired households, this is equivalent to drawing randomly from the distribution of newborns, but saves having to actually calculate such distribution.

the average ratio across remaining households is calculated.[56]

Intergenerational correlation of earnings. This is measured as the correlation between the average annual earnings of two consecutive generations of the same dynasty. In order to compute this moment, a random newborn is drawn from the distribution of newborns in a first step (see footnote 55). Next, the household is simulated recording its annual earnings until it dies twice. Then, on the basis of this, the average annual earnings for the first and second generations of the dynasty are calculated. This process is done for a large number of households and then the correlation between average annual earnings of both generations is calculated.[57]

C.3 Calibration

As previously commented in the document, the setup of this model economy has 36 parameters. 7 of them are normalizations and other 7 are directly identified. This leaves 22 parameters, which are estimated using the Simulated Method of Moments. It means that the values of these 22 parameters are those that minimize the distance between 22 moments of the model and the same 22 moments for the Spanish economy.

This model is a general equilibrium setup. Note that since aggregate production is depicted by a Cobb-Douglas production function, and due to the assumption of perfect competition implying that the interest rate equals the marginal product of capital, it is possible to identify the interest rate in terms of K/Y, α, and δ.[58] Thus, given that one target is K/Y, and since α and δ are directly identified, the value that the interest rate must take in equilibrium can be calculated. This feature is exploited in the calibration process. By taking the interest rate as an input, the target value on K/Y becomes in effect the general equilibrium condition. In other words, if a large weight is placed on the K/Y moment, the general equilibrium condition is being stressed throughout the calibration process. This little trick avoids the need to loop over the calculation of general equilibrium conditions all along. After reaching a certain arbitrary level of convergence the calibration process can be considered as completed. Thereafter, the general equilibrium must be calculated, as explained in the following Subsection C.4, with the calibrated parameters resulting from this process.

In the following, the implementation of the Simulated Method of Moments to estimate the 22 aforementioned parameters is detailed.

Step 1: A vector of 22 weights is chosen, one for each of the 22 moments. These weights measure the relative importance of each of the specified moments. In this sense, a greater weight

[56] Note that this should be done for a sufficiently large number of households since it must ensure that the simulation ends up with well in excess of households after dropping all of those households who retired.

[57] The number of households must be such that it ensures the stability of the statistic.

[58] $r = \alpha K^{\alpha-1}L^{1-\alpha} - \delta = \alpha \frac{1}{K/Y} - \delta$

is placed on the capital to output ratio, since, as commented before, this represents the condition of general equilibrium. In turn, the life cycle profile of earnings and the intergenerational correlation of earnings carry less weight, since neither of them is a clear map from the model to the data (the first due to stochastic aging and the second because what is really measured in the data is the correlation between parents and children). A greater weight is placed on the ratio of transfers to output, as this calibrates the parameter ω, namely the endowment of retirees, which plays a major role in the analysis. On the other hand, we should not put too much weight on G/Y, since it is defined as a leftover difference between Tr/Y and T/Y and is therefore closely related to other targets. Finally, the weights that the measures of inequality (distributional statistics) have are raised but not too much, since these parameters account for more than half of the total number of parameters.

Step 2: A guess is made for the values of the 22 parameter unknowns. At the time of implementing the guess, the initial values are based on the existing literature at first, and then they are based on previous runs of the code that were getting closer and closer to the level of convergence. This step took the most time.

Step 3: The Optimal policy function and the stationary household distribution are computed (given the interest rate and parameter values).

Step 4: The 22 moments of the model economy are computed (given the interest rate and parameter values).

Step 5: It checks whether the weighted distance of the model moments from the data moments is small enough. If the level of arbitrary convergence is reached, the calibration is completed. If not, new values are chosen for the parameters and return to step 3. (For this task the CMA-ES algorithm described below is used.)

Step 6: Once these parameters have been estimated, the general equilibrium of the model is calculated. (See Subsection C.4.)

The loop used to calibrate the parameters by matching the model moments to the data moments is implemented using the Covariance-Matrix Adaptation - Evolutionary Strategy (CMA-ES) algorithm. For a more involved explanation see Andreasen (2010), who also provides a Matlab code with the implementation of the algorithm. This algorithm outperforms many other inbuilt Matlab optimization functions (`fgoalattain`, `fminsearch`, `fminunc`, `fmincon`, among others) in the particular task of calibrating this model. This algorithm works by starting out considering the entire parameter space. Parameter vectors are drawn randomly (based on the covariance-matrix and a focal-point) and evaluated. Based on these evaluations the covariance-matrix and focal-point are updated. As the algorithm progresses the average distance between the parameter vectors drawn and the focal-point is progressively reduced. Once certain convergence criterion is met, the focal-point is returned as the estimated value of the true parameter vector. For running this subroutine, the lower and upper bounds of the parameter space and the size of the step in the parameter search must be specified.

C.4 General Equilibrium

The calculation of a general equilibrium in this family of models typically involves finding an interest rate which induces individual behavior which generates aggregate variables (like output, labor, or capital) that in turn lead back to the original interest rate. A prominent example of this is Aiyagari (1994). In this model economy, in addition to the market clearance condition (given by the interest rate), it is also required that the government balances its budget to determine the general equilibrium. These conditions result in two requirements (the so called general equilibrium equations): (i) interest rate equals the marginal product of capital ($r = \alpha K^{\alpha-1} L^{1-\alpha} - \delta$) and (ii) government balances its budget ($G/Y + Tr/Y = T/Y$).

In order to find the solution to these two general equilibrium equations, two price solvers are needed. These prices are the interest rate, r, and the linear term on remaining taxes, κ.[59] In this sense, we use the general equilibrium finding stage as the calibration procedure for this parameter κ, which is previously determined by the calibration algorithm, but here it is where it takes its final calibrated value so that the government in the model economy runs a budget-balance fiscal policy. Further, in comparison with the traditional methods, there is a noteworthy difference in the way that this algorithm finds the interest rate that ensures market clearance. Here, rather than using a search algorithm on K/Y to find the general equilibrium interest rate (the standard approach), the algorithm discretizes the state space for r and use this to find the equilibrium value. The same procedure is applied on κ.

In summary, this algorithm calls the Matlab `fminsearch` subroutine, which finds the values of r and κ that respectively minimize the previously described equations. Once a certain level of arbitrary convergence has been reached, it can be said that the algorithm has found the general equilibrium.[60]

[59] Note that κ does not explicitly enter the second general equilibrium equation, but its role is already captured in the aggregate variables.

[60] Many of the optimization algorithms normally applied for this step rely on differentiability and convexity for convergence, neither of which is known to hold. They also assume that the solution found is a global solution, rather than simply local solution. Here, this algorithm relies on the initial values for r and κ that come from the calibration algorithm. It means that the general equilibrium is found in the closest solution to such values, which does not ensure that the solution is global.

C.4.1 General Equilibrium in A Reformed Economy

For each reform, i.e. in this study, for each level of τ, a new general equilibrium needs to be calculated. To do this, the same method that was presented in the lines above is used. The only slight change is that the parameter κ is no longer a price solver of the second general equilibrium equation anymore. Now, as each reform requires the government to keep its budget balanced and to guarantee the same level of government expenditure, G/Y, the parameter that will serve as a price solver of the second general equilibrium equation will be ω. This is where the redistributiveness of setting a different τ is evaluated in terms of increments or decreases in pensions. Beyond this, the computational procedure for the search of a general equilibrium for each new scenario is the same as the one presented above.

Declaration of Academic Integrity

"I, Darío Serrano Puente, hereby declare that this master's thesis is entirely my own work, and that I have employed no sources or aids other than the ones listed. I have clearly marked and acknowledged all ideas that have been taken directly or indirectly from the works of others. I also confirm that this thesis has not been submitted in this form, or a similar form, to any other academic institution."

Darío Serrano Puente
Mannheim, 27.01.2020